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KLT-DETECTED TRANSIENT SIGNALS FROM RELATIVISITIC SPACESHIPS

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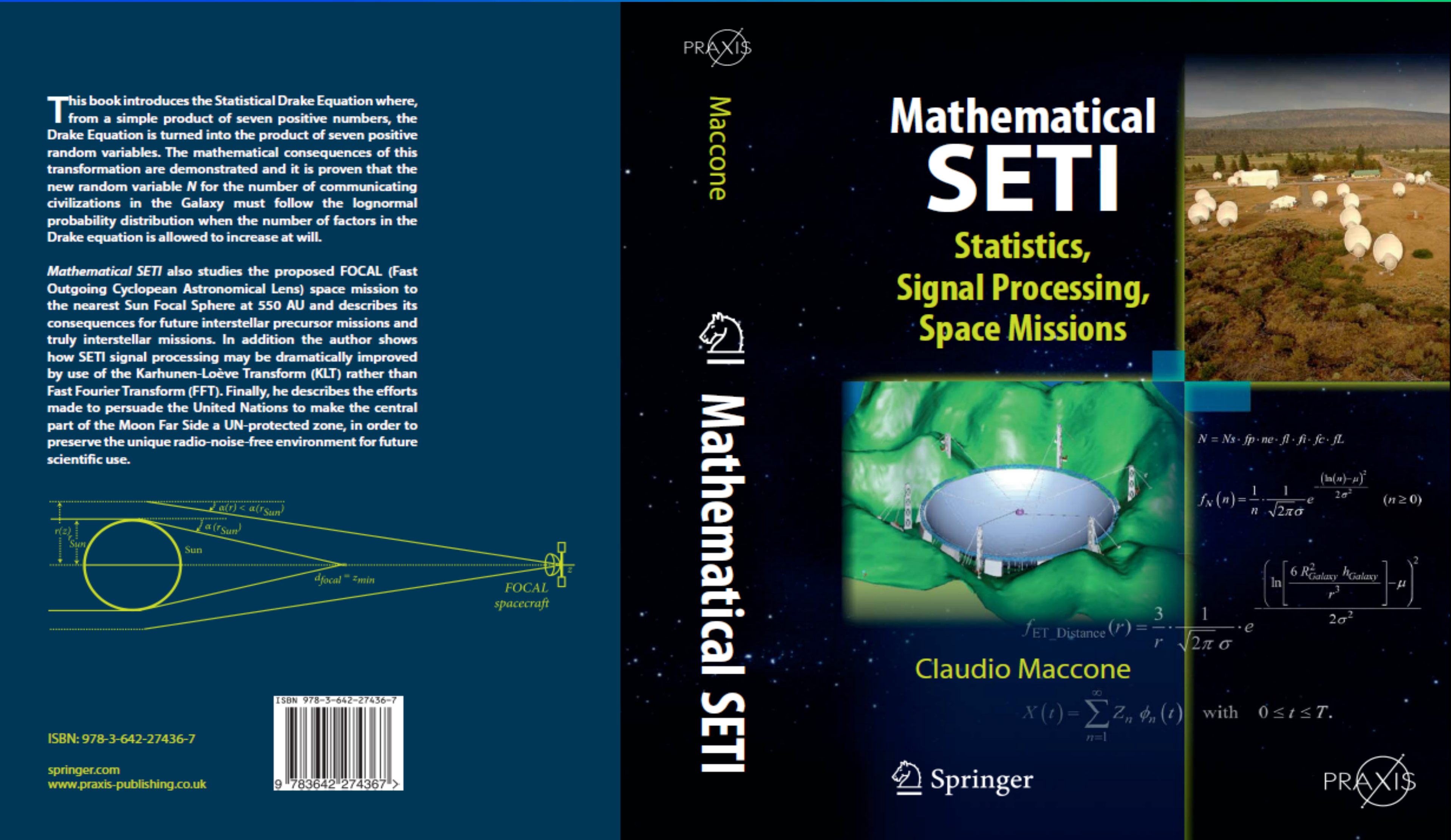
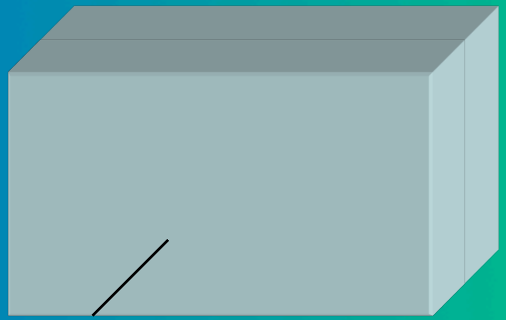
700-pages BOOK about "Mathematical SETI" (2012)



What is the KLT ?

- Example (a Newtonian analogy): consider a solid object, like a BOOK, described by its INERTIA MATRIX.

- Then there exist only one special reference frame where the Inertia Matrix is DIAGONAL. This is the reference frame spanned by the EIGENVECTORS of the Inertia matrix.



$$I' = \begin{pmatrix} I'_{xx} & 0 & 0 \\ 0 & I'_{yy} & 0 \\ 0 & 0 & I'_{zz} \end{pmatrix}$$

KLT mathematics

- If $X(t)$ is a stochastic process (= input to the radio telescope) it can be expanded into an infinite series

$$X(t) = \sum_{n=1}^{\infty} Z_n \, \phi_n(t) \qquad 0 \le t \le T$$

- then
  - $\phi_n(t)$ are orthonormalized functions of the time:

$$\int_0^T \phi_m(t)\, \phi_n(t)\, dt = \delta_{mn}$$

Z_n are random variables, not changing in time, with the property

$$\langle Z_m \, Z_n \rangle = \lambda_n \, \delta_{mn}$$

- In conclusion, the KLT separates the radiotelescope input (= noise + signal(s)) into ***UNCORRELATED*** components.

KLT mathematics

$$\int_0^T \langle X(t_1)\, X(t_2) \rangle \; \phi_n(t_1) \; dt_1 = \lambda_n \; \phi_n(t_2)$$

- This is the integral equation yielding the Karhunen-Loève's eigenfunctions $\phi_n(t)$ and corresponding eigenvalues λ_n .
- The kernel of this integral equation is the autocorrelation.
- This is the best basis in the Hilbert space describing the (signal +noise). The KLT adapts itself to the shape of the radiotelescope input (signal+noise) by adopting, as a reference frame, the one spanned by the eigenfunctions of the autocorrelation. And this is turns out to be just a LINEAR transformation of coordinates in the Hilbert space. Thus, the KLT is an easily INVERTIBLE TRANSFORMATION.

KLT Filtering

- There is no degeneracy (i.e. each eigenvalue corresponds to just one eigenfuction only).

- The eigenvalues turn out to be the variances of the random variables Z_n , that is $\sigma_{Z_n}^2 = \lambda_n = \langle Z_n^2 \rangle > 0.$

- Since $\langle Z_n \rangle = 0$ we can SORT in descending order of magnitude both the eigenvalues and the corresponding eigenfunctions. Then, if we decide to consider only the first few eigenfunctions as the "bulk" of the signal, and to apply the inverse KLT, that's what KLT filtering is: we just declare the taken-off part as "noise"!

- The Galileo mission by NASA-JPL used the KLT…

"Classical" KLT vs. FFT

	KLT	FFT	
↑	Works well for both wide and narrow band signals	Rigorously true for narrow band signals only	↓
↑	Works for both stationary and non-stationary input stochastic processes	Works OK for stationary input stochastic processes only	↓
↑	Is defined for any finite time interval	Is plagued by the "windowing" problems	↓
↓	Needs high computational burden: no "fast" KLT	Fast algorithm FFT	↑

NO Classical KLT in the1990s

- If N is the size of the autocorrelation matrix (N may equal millions ore more in SETI), the number of calculations requested to find the KLT is of the order of N square, while the same number for the FFT is much less: just N ln(N).

- This ***COMPUTATIONAL BURDEN*** prevented all SETI scientists from replacing the FFT by the KLT until 2007:

1) François Biraud et al. at Nançay in the years after 1983.

2) Bob Dixon et al. the Ohio State SETI Program after 1985.

3) Stelio Montebugnoli et al. at Medicina, Italy, after 1990.

BAM (=Bordered Autocorrelation Method) to EASILY find the KLT

- BAM is acronym for "BORDERED Autocorrelation Method".

- The new key idea is to regard the autocorrelation as a NEW FUNCTION OF KLT FINAL INSTANT, *T*. That is, to add one more row and one more column (= bordering) to the autocorrelation matrix for each new positive, increasing *T*.
- For STATIONARY processes, this amounts to the matrix:

$$R_{Toeplitz} = \begin{bmatrix} R_{XX}(0) & R_{XX}(1) & R_{XX}(2) & \cdots & \cdots & R_{XX}(N) \\ R_{XX}(1) & R_{XX}(0) & R_{XX}(1) & \cdots & \cdots & R_{XX}(N-1) \\ R_{XX}(2) & R_{XX}(1) & R_{XX}(0) & \cdots & \cdots & R_{XX}(N-2) \\ \cdots & \cdots & \cdots & R_{XX}(0) & \cdots & \cdots \\ R_{XX}(N) & R_{XX}(N-1) & \cdots & \cdots & R_{XX}(1) & R_{XX}(0) \end{bmatrix}$$

2007 Breakthrough in the KLT

- In the winter of 2006-7, the SETI-Italia Group at Medicina, (Stelio Montebugnoli, Francesco Schillirò, Salvo Pluchino and Claudio Maccone) discovered a way to CIRCUMVENT that big obstacle of the KLT N^2 computational burden.

- The idea is to use the BAM to exploit the **<u>dependence on the final instant</u>** T in both sides of the relationship, firstly proved in 1994 by Maccone in his KLT book, p.12, eq. (1.13):

$$\sum_{n=1}^{\infty} \lambda_n(T) = \int_0^T \sigma^2_{X(t)}\, dt$$

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- Differentiating both sides wrt T yields the FINAL VARIANCE THEOREM (Maccone, Proc. of Science, 2007)

$$\sum_{n=1}^{\infty} \frac{\partial \lambda_n(T)}{\partial T} = \sigma^2_{X(T)}$$

- Confining ourselves to the FIRST EIGENVALUE (the "dominant one" = the largest) & STATIONARY ***X(t)*** ONLY:

$$\sum_{n=1}^{\infty} \frac{\partial \lambda_n(T)}{\partial T} \approx \frac{\partial \lambda_1(T)}{\partial T} = \sigma^2_X = \text{a\_CONSTANT\_wrt\_T}$$

- The SETI-Italia Team discovered that ***the Fourier transform of this constant is a peak (i.e. Dirac delta function) that is the FREQUENCY of the ET-SIGNAL !!!***

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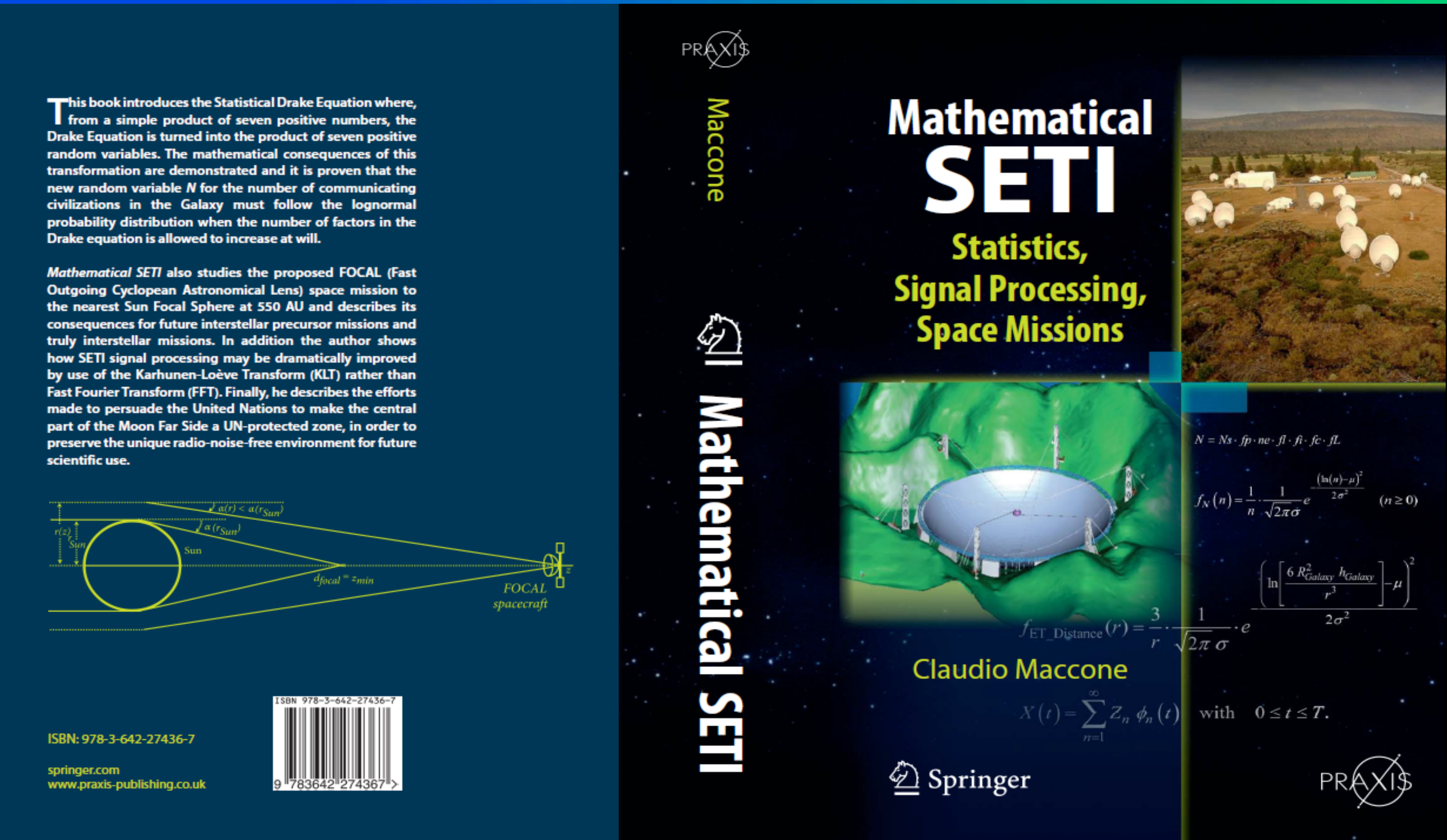

Thanks !