(Self-)calibration: from knowing everything to knowing nothing at all

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A quote from Wim Brouw

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Self-calibration uses the observed data to calibrate themselfs. At first sight this sounds like lifting yourself out of quicksand by pulling your hair. However, two properties make this feat possible:

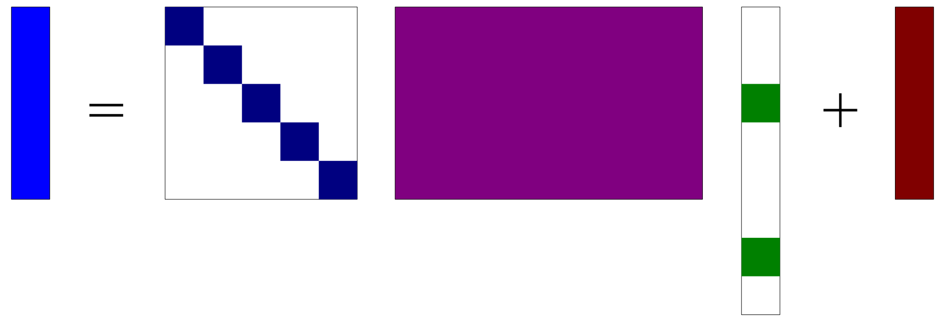
– most observing errors occur on a per array-element (telescope) basis: atmospheric disturbances are above one telescope, receiver instabilities are for receivers in one telescope (or at least decoupled from similar instabilities in other telescopes).
– even an at first sight crowded sky field is mostly devoid of radiation, making it possible to model the sky with a limited number of source components

Source: W. N. Brouw, "The synthesis radio telescope: principles of operation; evolution of data processing." In: E. Raimond and R. Genee (eds.), "The Westerbork Observatory, Continuing Adventures in Radio Astronomy," Kluwer, 1996.

Exploiting sparsity (1)

Data model (ME) with a “limited number of source components”

$$\mathbf{x}(t) = \mathbf{G} \quad \mathbf{A} \quad \mathbf{s}(t) + \mathbf{n}(t)$$



Voltage / signal domain solution proposed and demonstrated

- Kazemi et al., IEEE ICASSP, June 2015

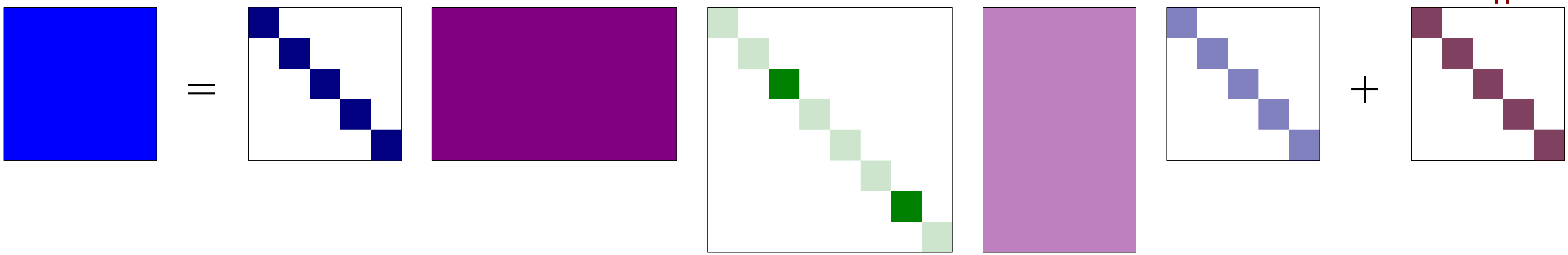
Minor detail: calibration on 1 s of data for a single 195 kHz subband of LOFAR may require 10^7 Yflop (1 Yottaflop = 10^{24} flop)

Conceptually nice, but we may need some speed-up here …

Exploiting sparsity (2)

Data model (ME) in power / visibility domain

$$\mathbf{R} = \mathbf{G}\,\mathbf{A}\,\mathbf{\Sigma}\,\mathbf{A}^H\,\mathbf{G}^H + \mathbf{\Sigma}_n$$



Problem is non-convex, but can be solved iteratively

- Wijnholds & Chiarucci, EuSiPCo, August 2016

Compute requirements for calibration of a single subband of LOFAR data reduced from 10^7 Yflop to 10 Gflop (station correlator: ~ 1 Gflop)

Application to LOFAR

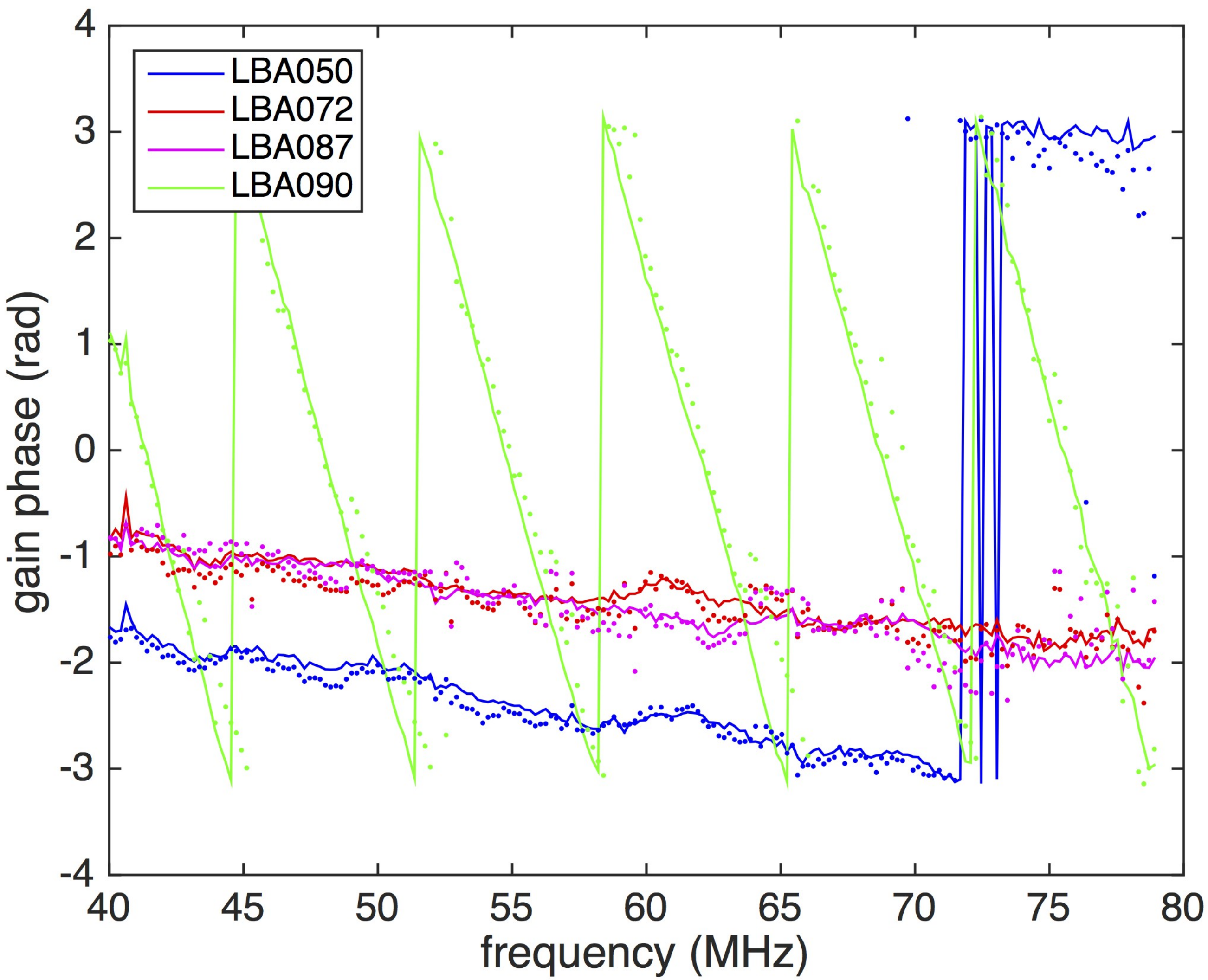
Chiarucci & Wijnholds, MNRAS, under review

gain phase solutions from blind calibration and standard calibration

CS002 LBA outer

Subbands 205 – 45
(40.0 – 78.9 MHz)

Coarse alignment
(Δl,Δm = 0.0185)



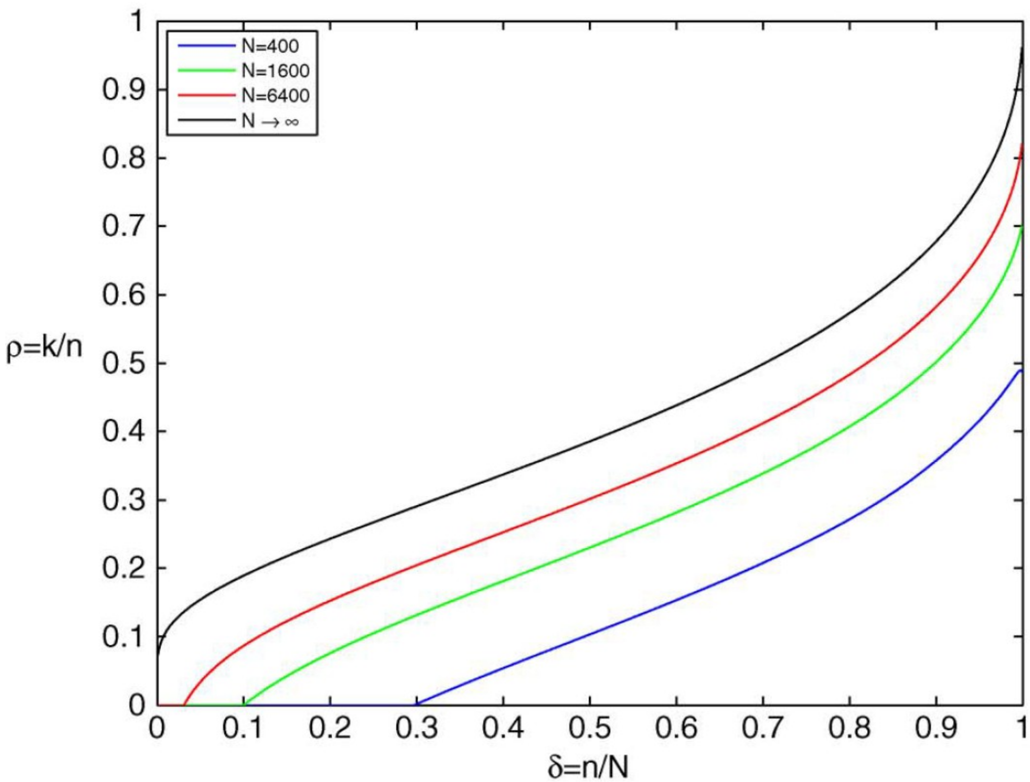
The phase transition diagram
Donoho & Tanner, Proc. IEEE, June 2010

ρ (sparsity factor): #components / #measurements

δ (undersampling factor): #measurements / #parameters

DT-curve: 50% chance of successful reconstruction

DT-curve shifts towards lower right with decreasing problem size



Probability of success

Chiarucci & Wijnholds, MNRAS, under review

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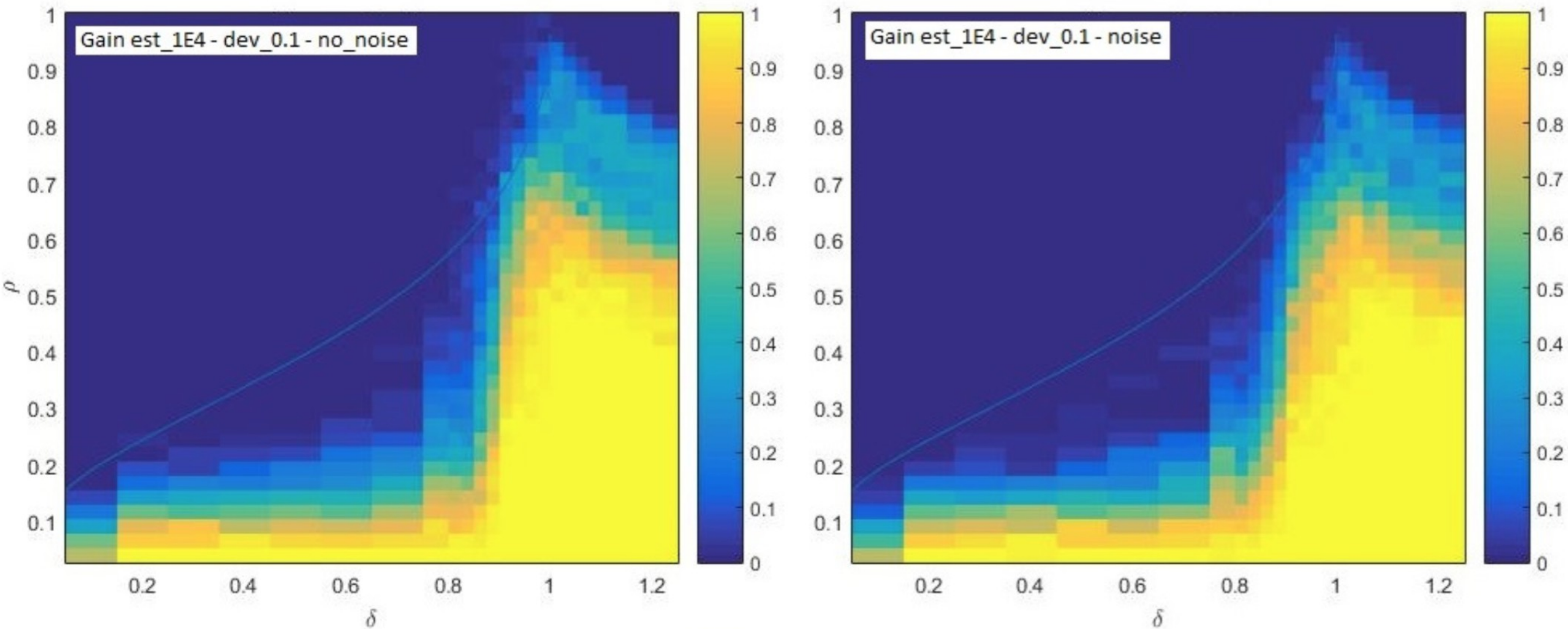
Simulation for 20-element ½λ-spaced Uniform Linear Array

ρ (sparsity factor): #sources / #unique visbilities (39)

δ (undersampling factor): #unique visibilities (39) / #image grid points



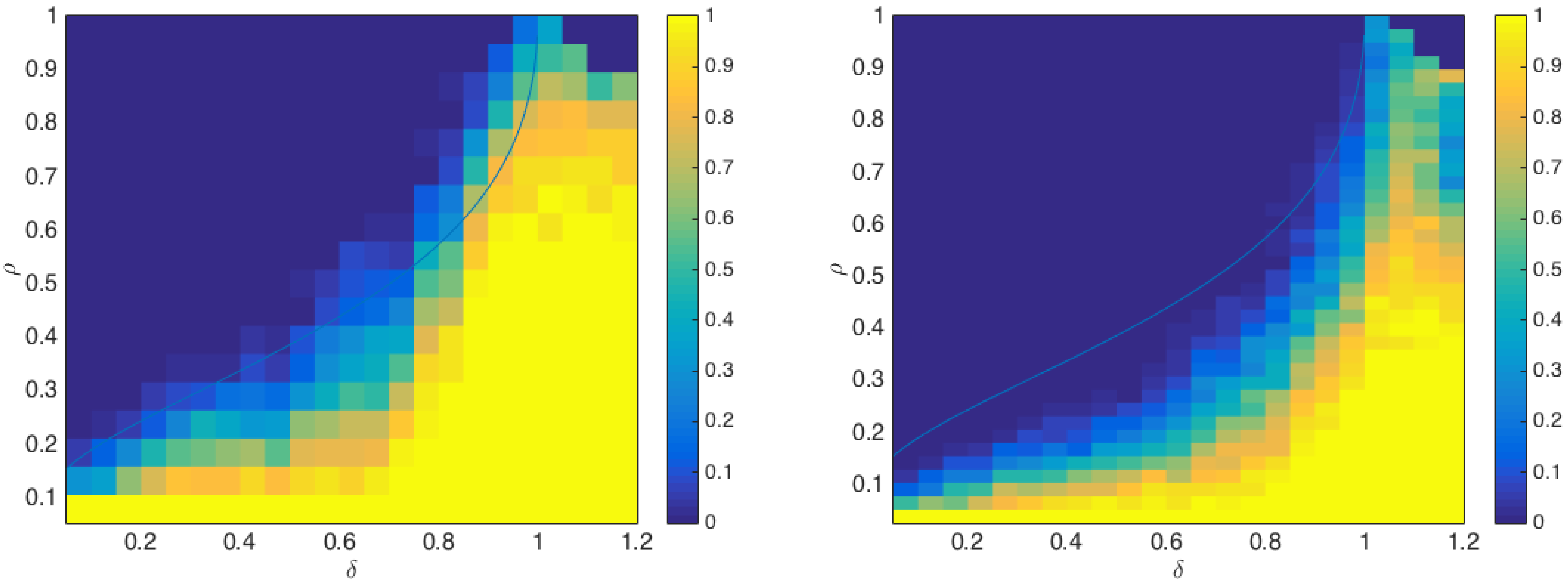
Impact of redundancy

Chiarucci & Wijnholds, MNRAS, under review

Left: phase diagram for minimum redundant array

Right: phase diagram for irregular array

Lower redundancy brings us closer to DT curve



Implications for self-cal

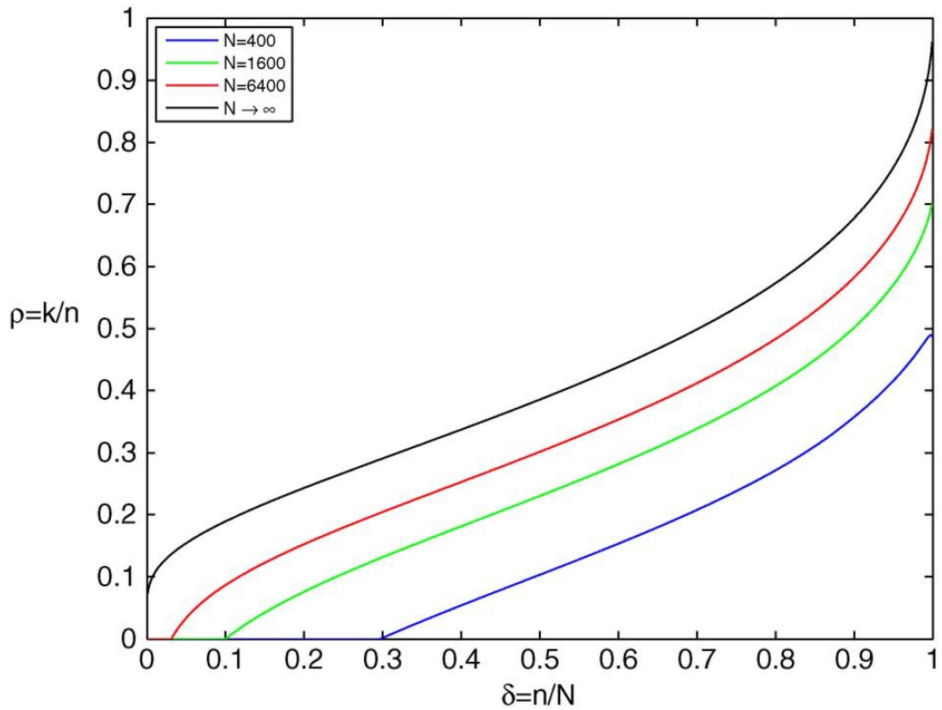
Chiarucci & Wijnholds, MNRAS, under review

Observations:

- If map is not confusion limited, $\rho < 0.1$
- Synthesis observations provide good (u,v)-coverage: $\delta > 0.2$
- Large problem size: $N \sim 10^6 - 10^9$

Blind calibration with sparsity constraint is almost sure to work.

Self-calibration should be able to recover from poor initial estimate.



A closer look at the LOFAR result

Chiarucci & Wijnholds, MNRAS, under review

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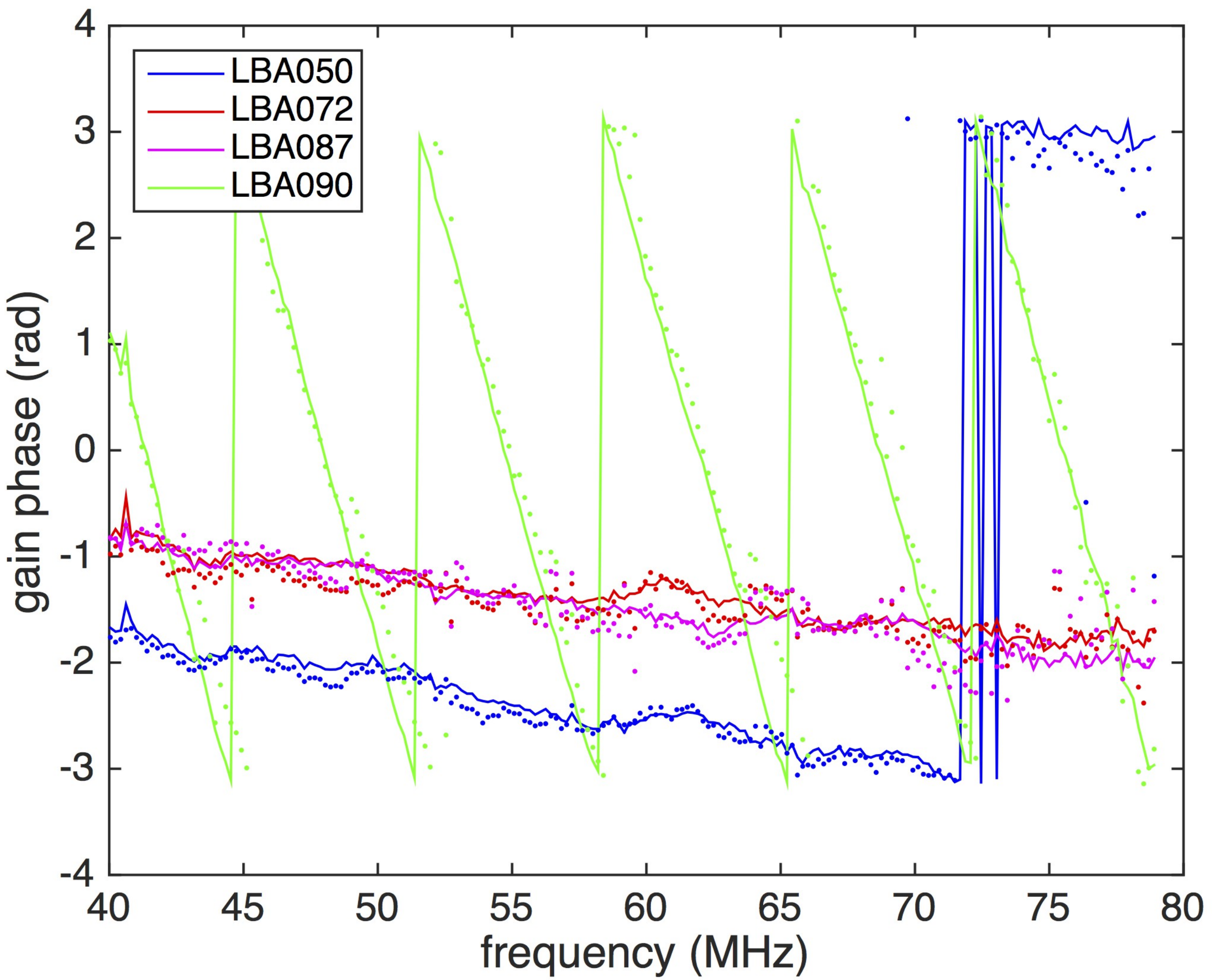

Accuracy of blind calibration

Ben-Haim & Eldar, IEEE TSP, 2010

Theoretical result

> If the source model is identifiable, the Cramer Rao bound for image reconstruction is identical to that of the oracle estimator

Consequence for self-cal

> The calibration accuracy achievable with blind calibration is identical to the accuracy achievable in calibration with DDEs common to all receivers

Conclusion

The recently developed theory of compressive sampling provides a tool to quantitatively understand the empirical (and sometimes surprising) self-calibration results from the past.