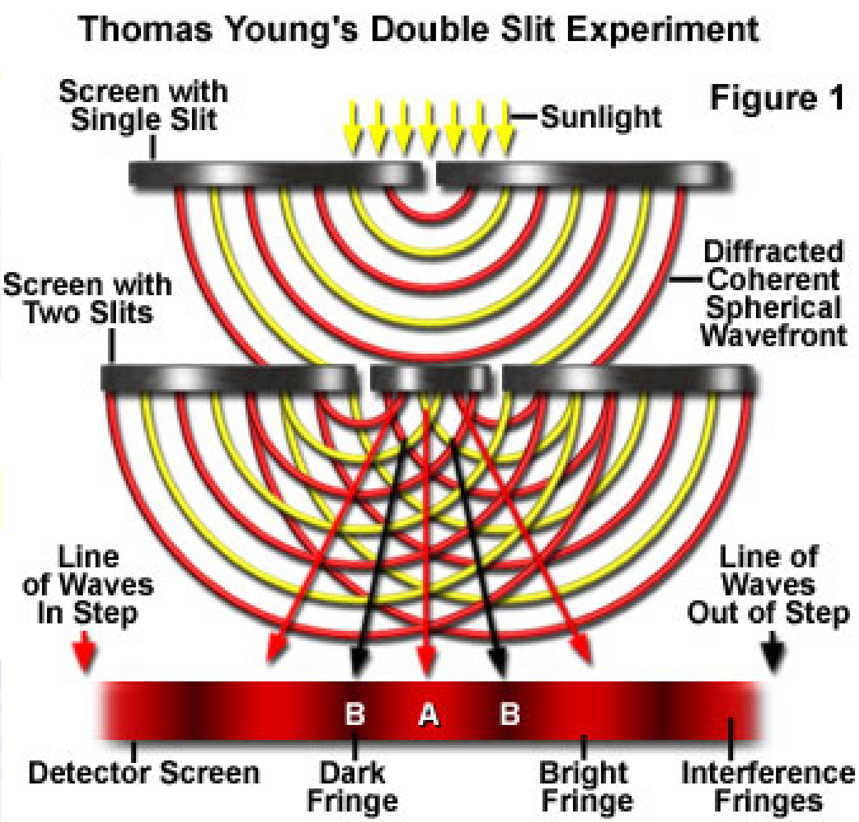
Lectures on radio astronomy: 4

Richard Strom
NAOC, ASTRON and University of Amsterdam

Interferometers

Early interferometry: Young's double slit experiment (1801)



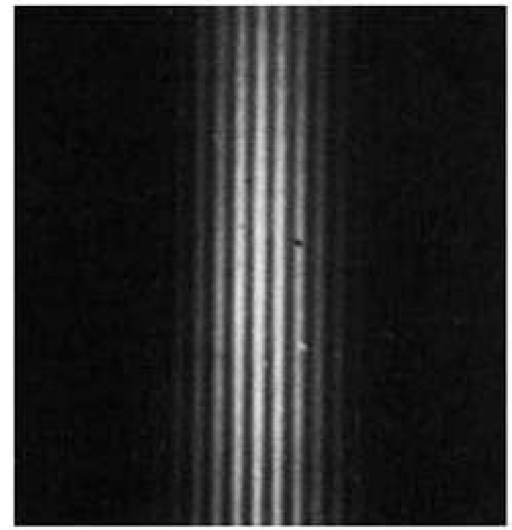


Image B

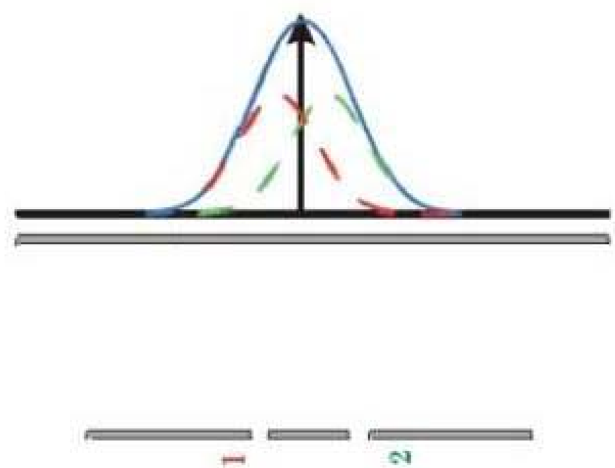


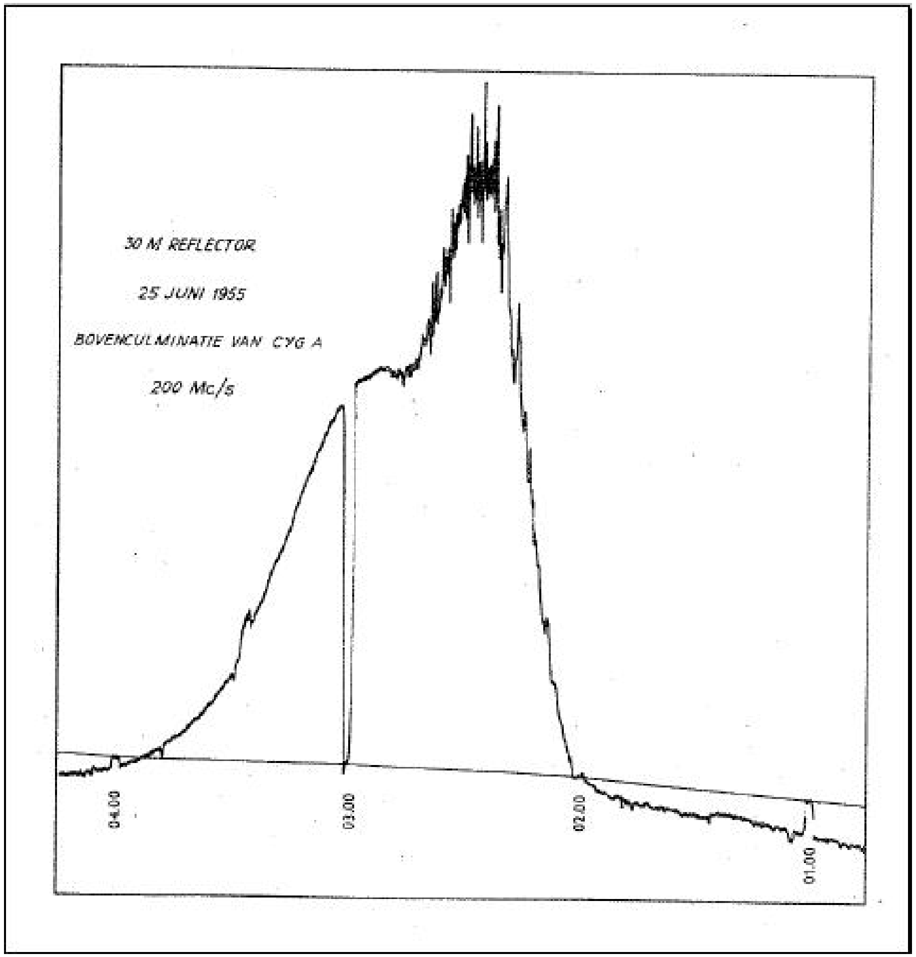
Image A

Radio engineers experimented in 1930s with interferometers

- A fixed array like the one shown here is a kind of interferometer
- Eight elements give a narrower beam
- It was originally used for radar in Australia

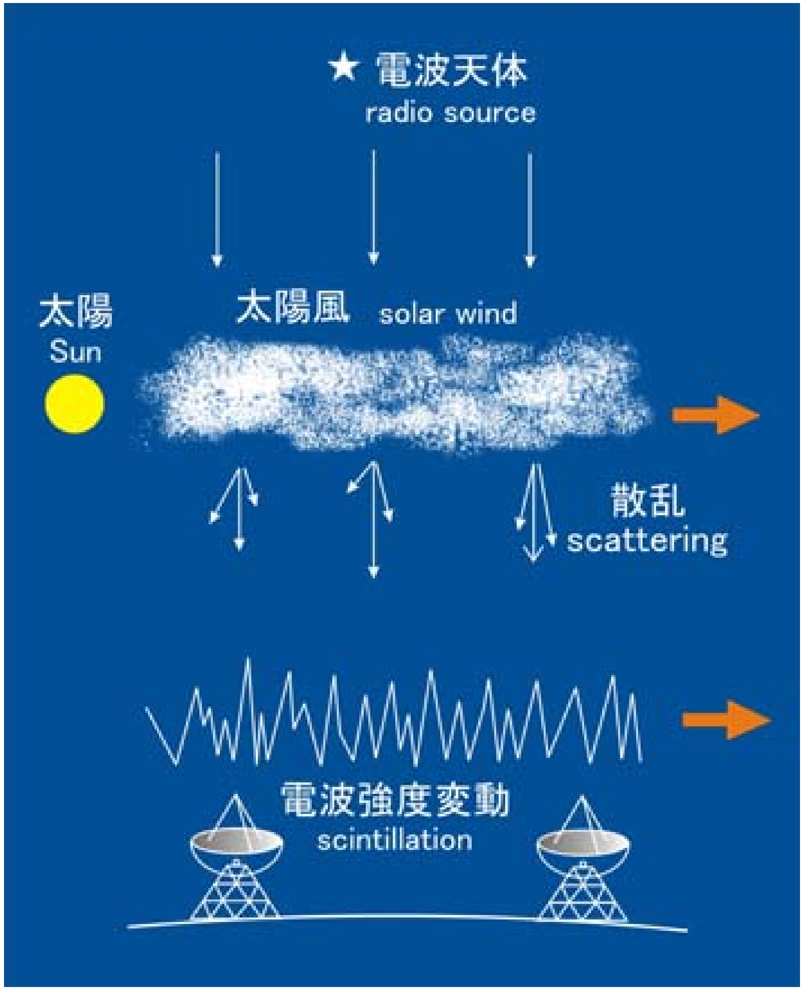
Radio telescopes in 1940s worked at long wavelengths

- Angular resolution quite limited
- Most sources were unresolved
- J.S. Hey noticed fluctuations in signal from Cygnus A
- Concluded must be scintillation in ionosphere & source must be compact



Scintillation – schematic description

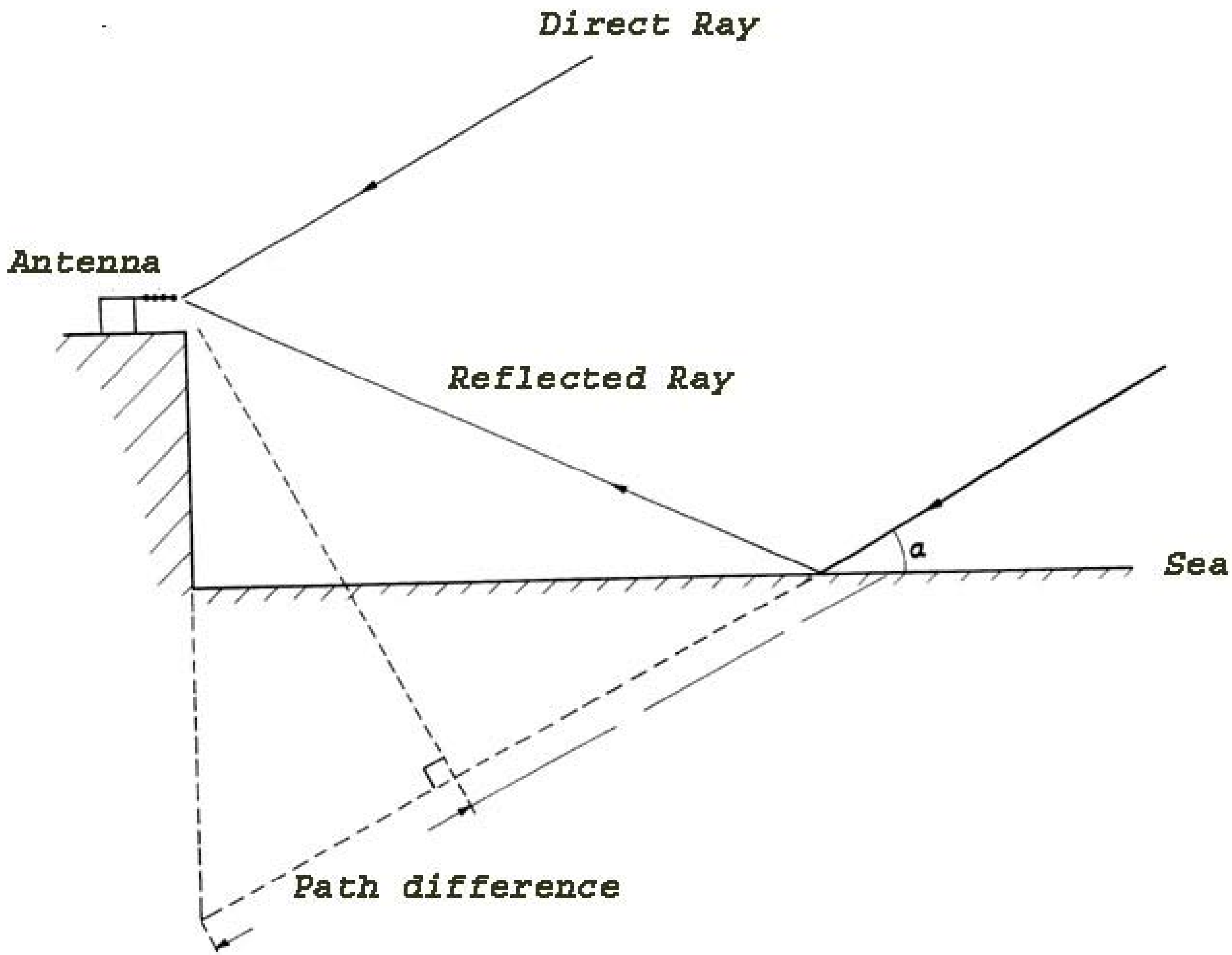
- Undistorted plane wave reaches ionized region
- Irregularities cause scattering by refraction
- Original wavefront now has brighter and fainter regions
- Scintillation can also be seen as a form of interferometry



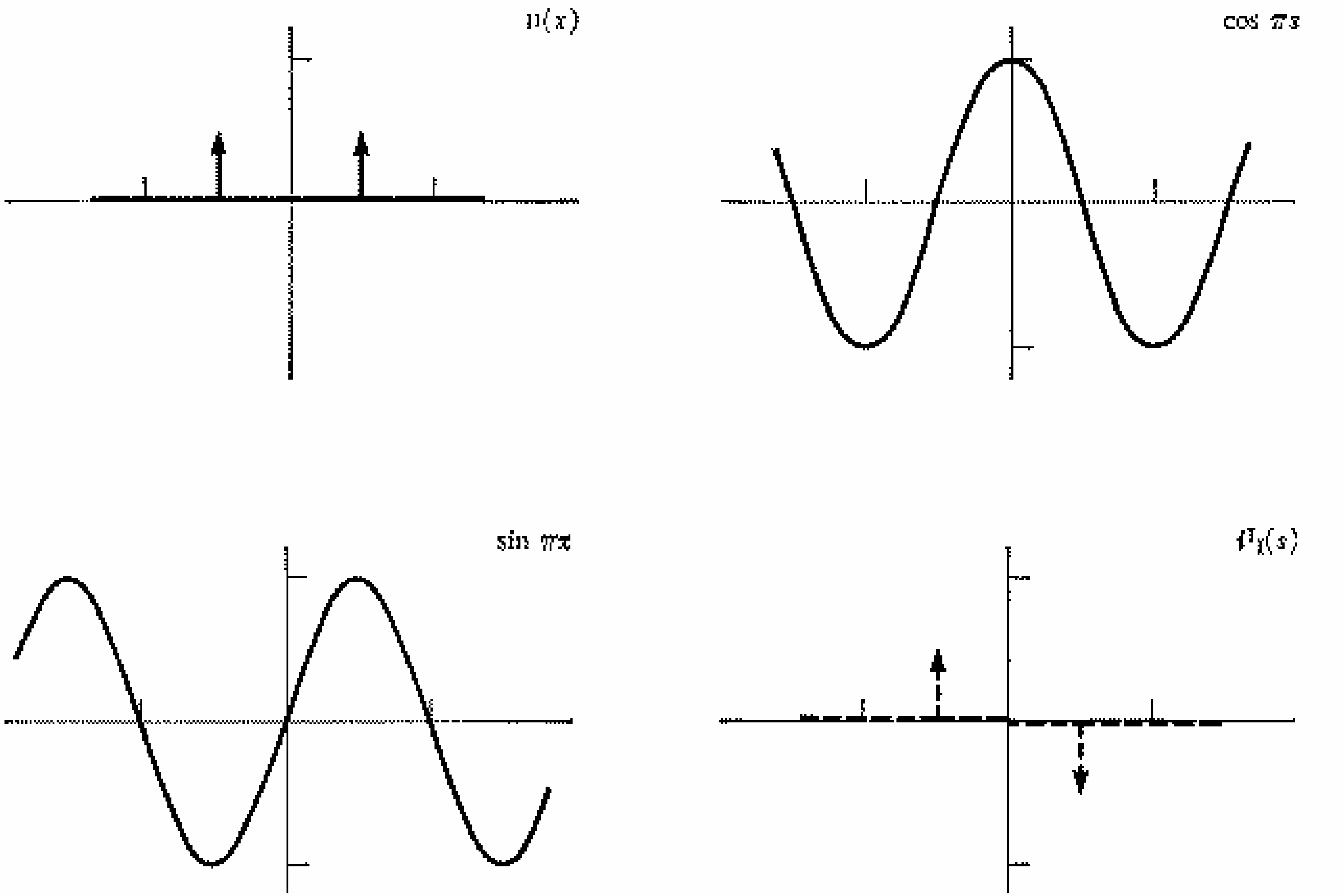
Early interferometry with one antenna

Geometry of Australian cliff-top antenna

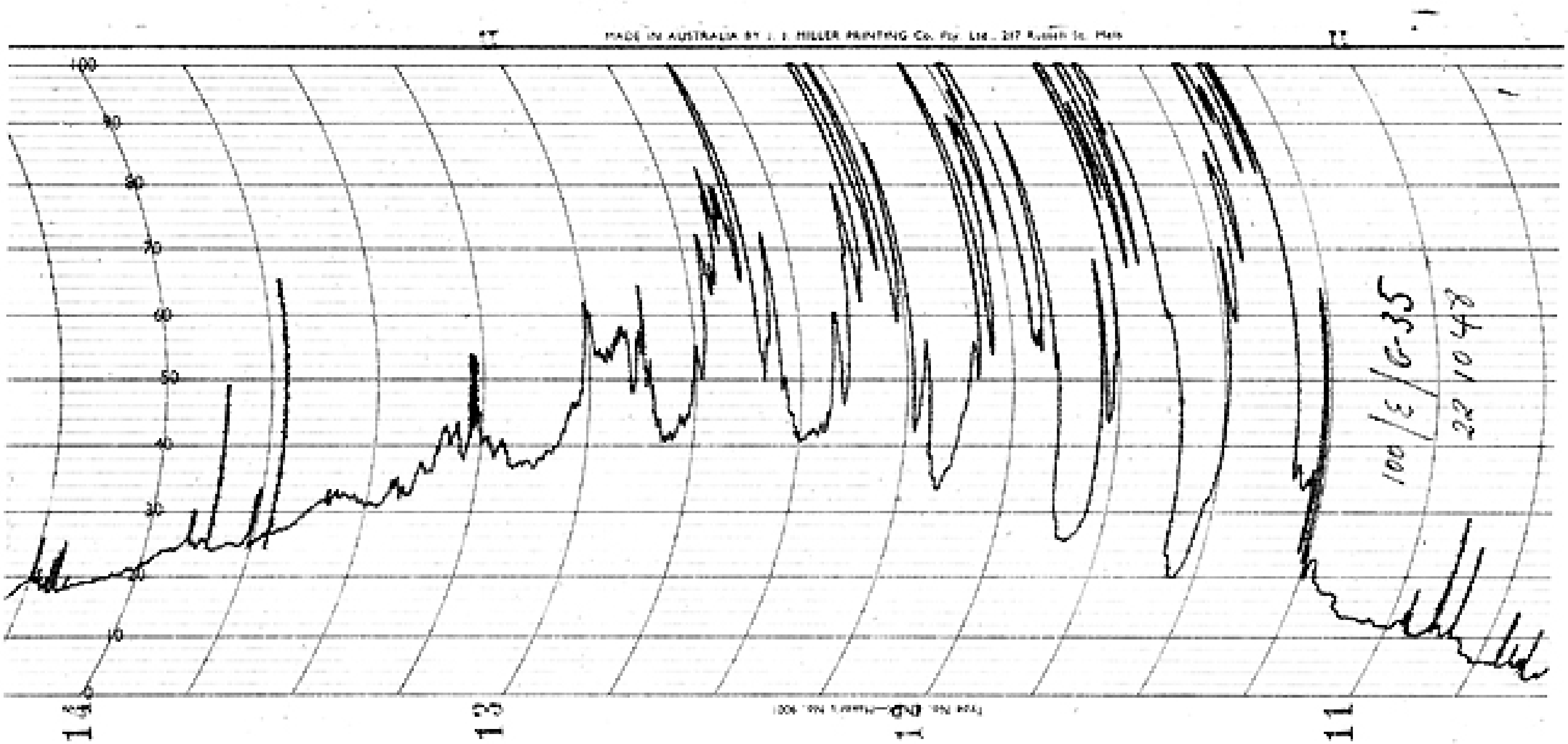
(in optics, called Lloyd's mirror)



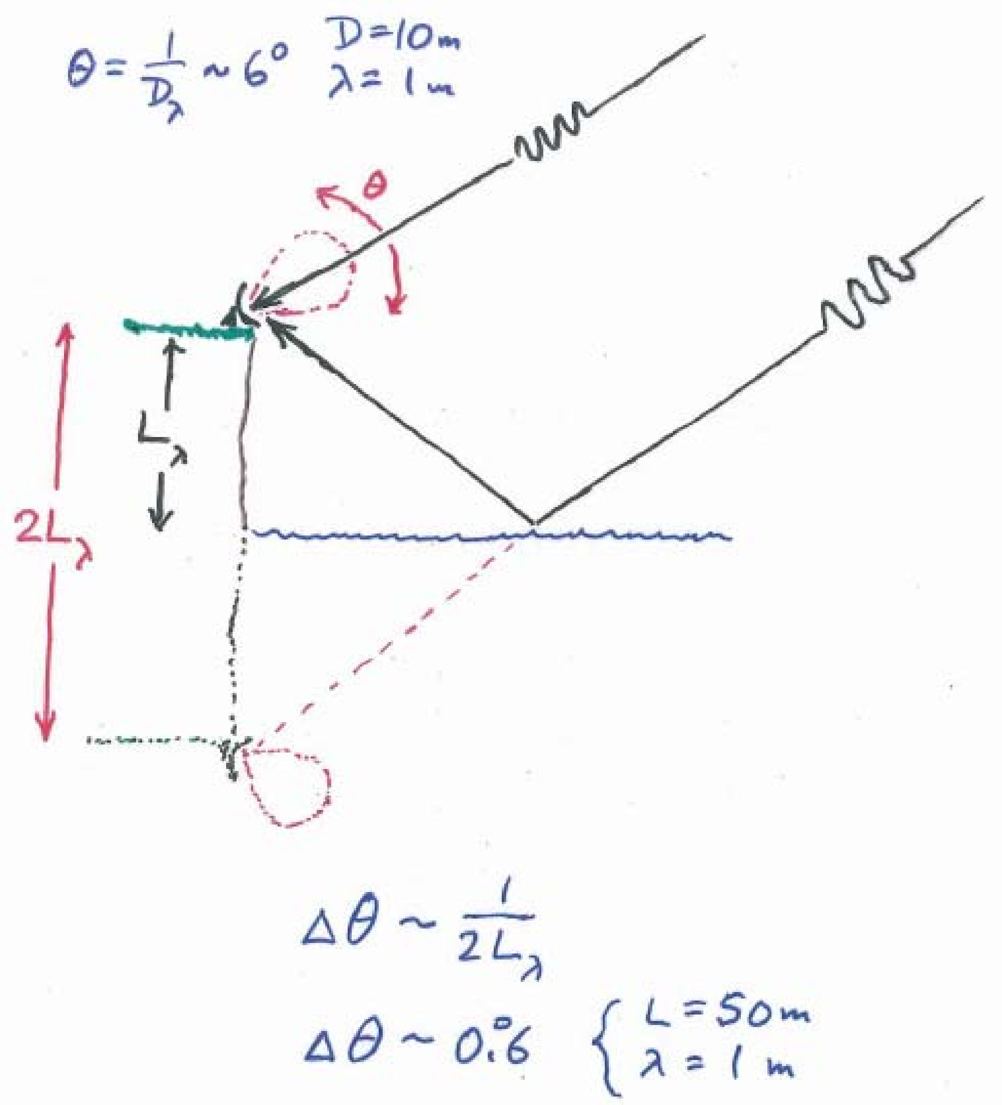
Response of two elements found by Fourier transform



Observation of Cygnus A – note cosine & scintillation



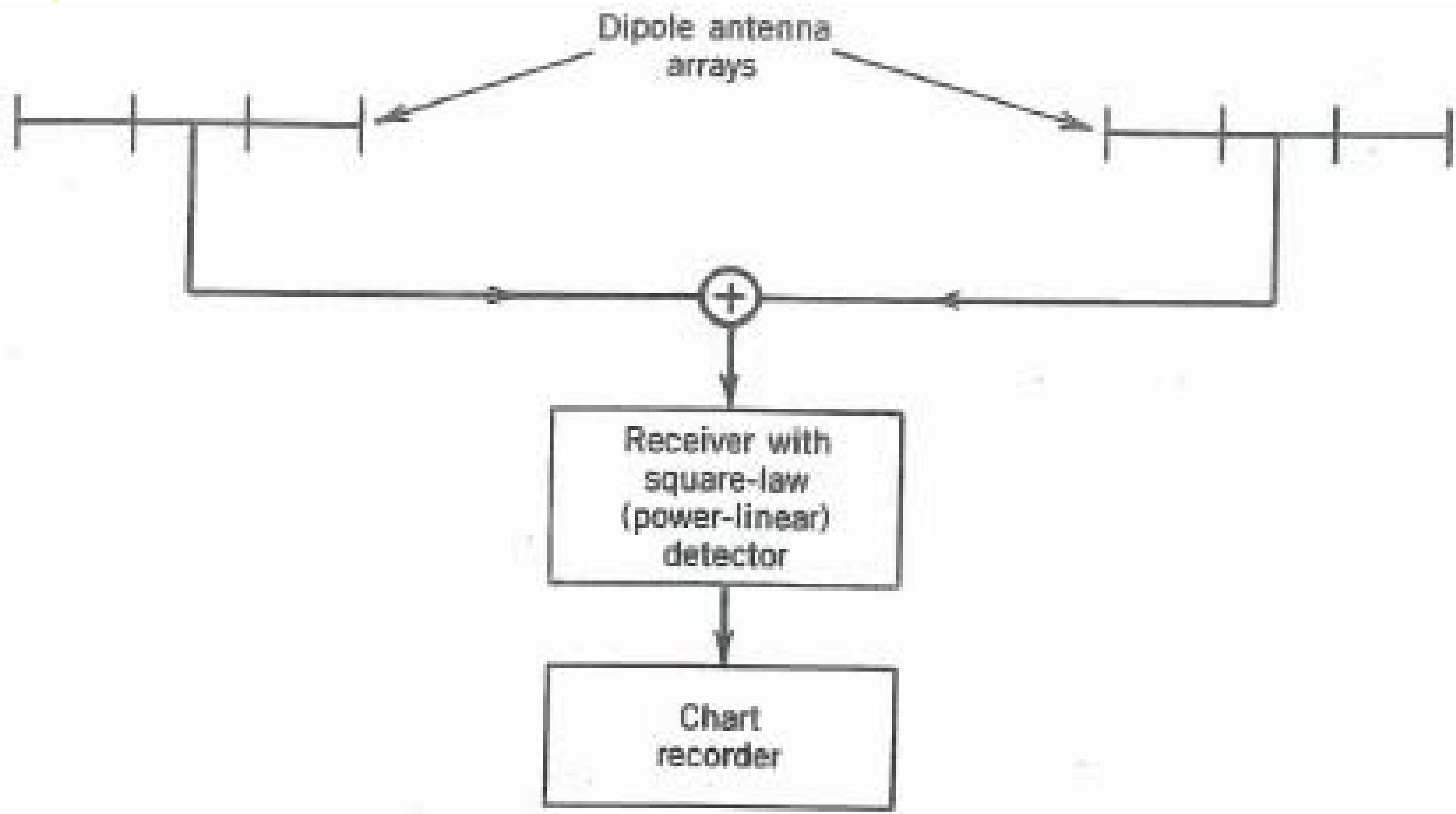
Angular resolution of the cliff-top interferometer



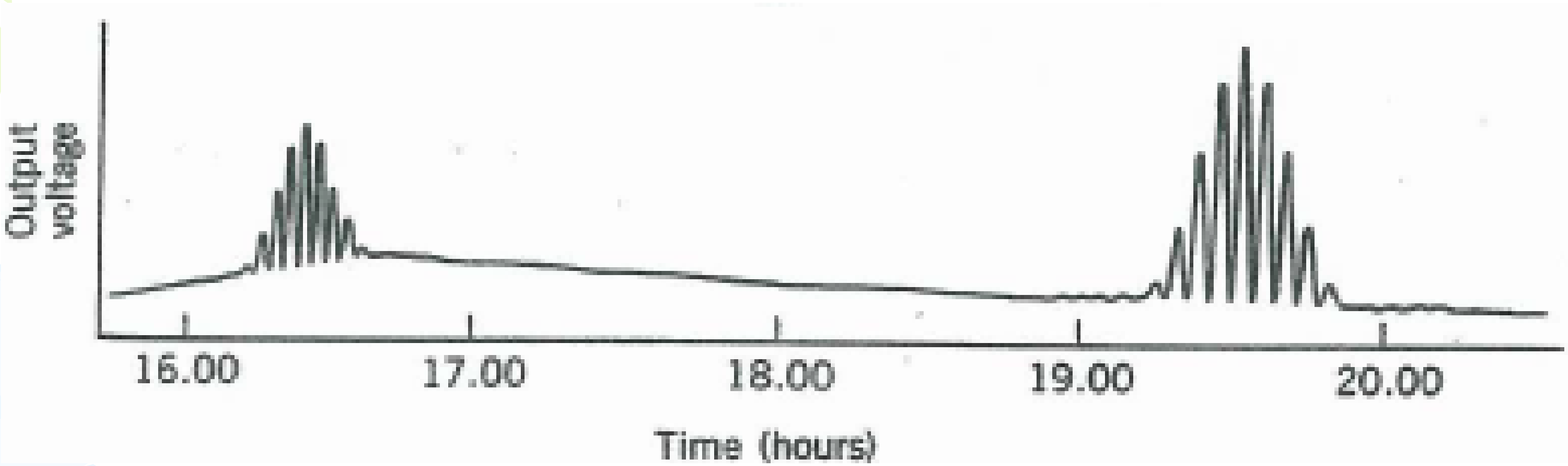
The cliff-top interferometer was a clever idea, but…

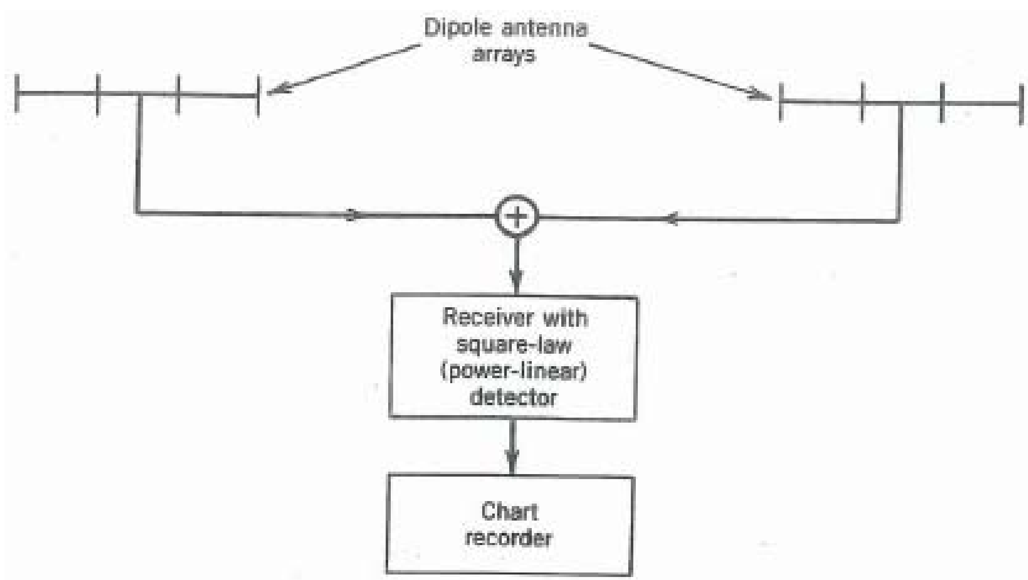
- Sources could only be observed at low elevation
- Ionosphere much thicker – sec z effect
- Refraction and scintillation made interpretation difficult

People also experimented with 2-element interferometers

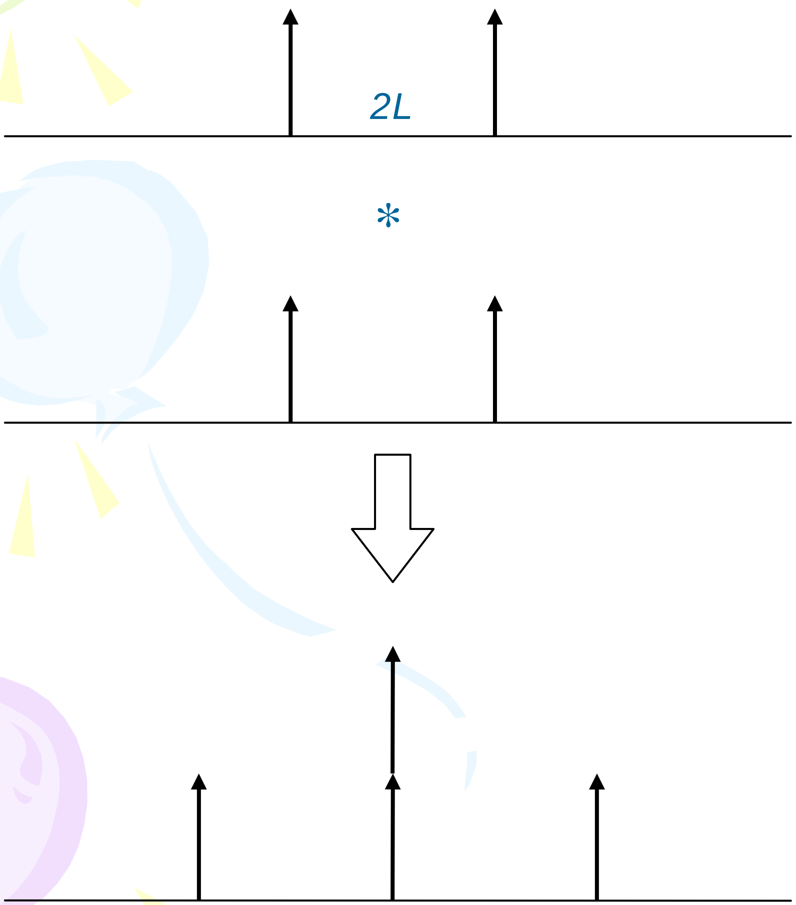


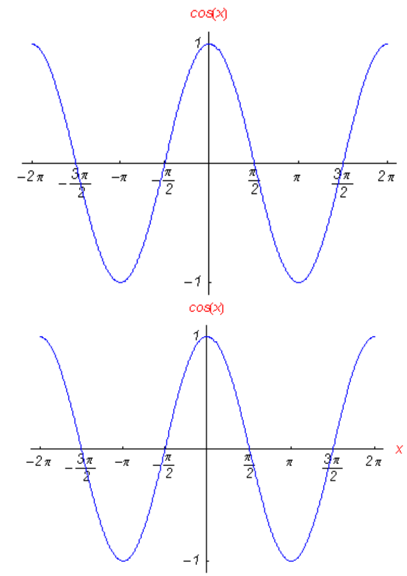
And this would be the result: beam * source

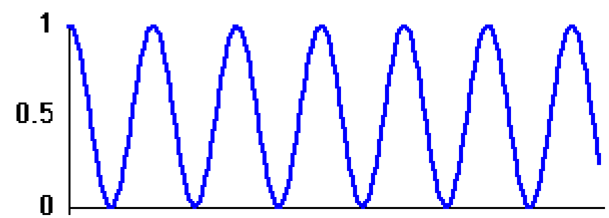




Fourier analysis of simple 2-element interferometer

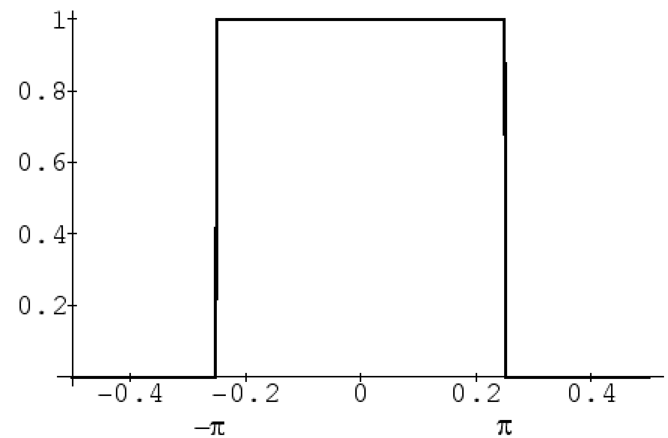


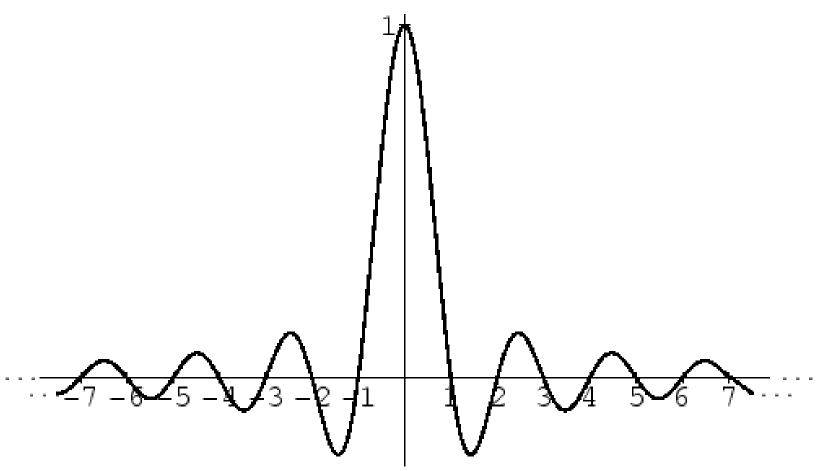




What happens if elements have their own beams?

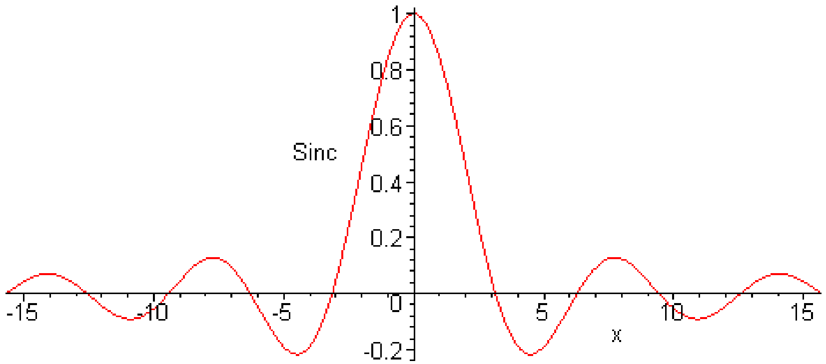
- As we know, for uniform illumination (square function)...
- ...the FT is a sinc (sin*x*/*x*) function.
- This is the voltage beam pattern. As we know, the power beam can be found from (voltage)^2 ...

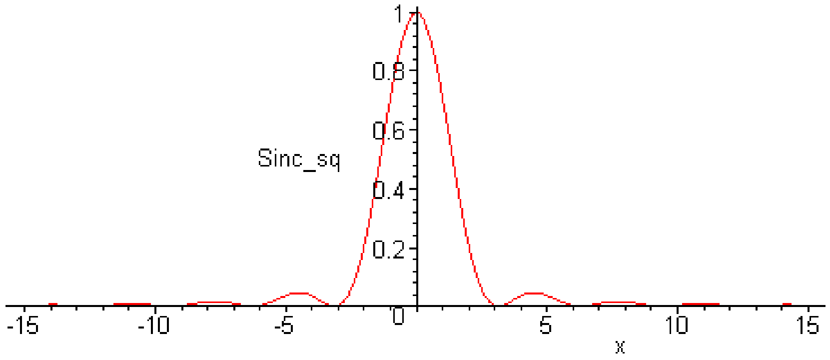




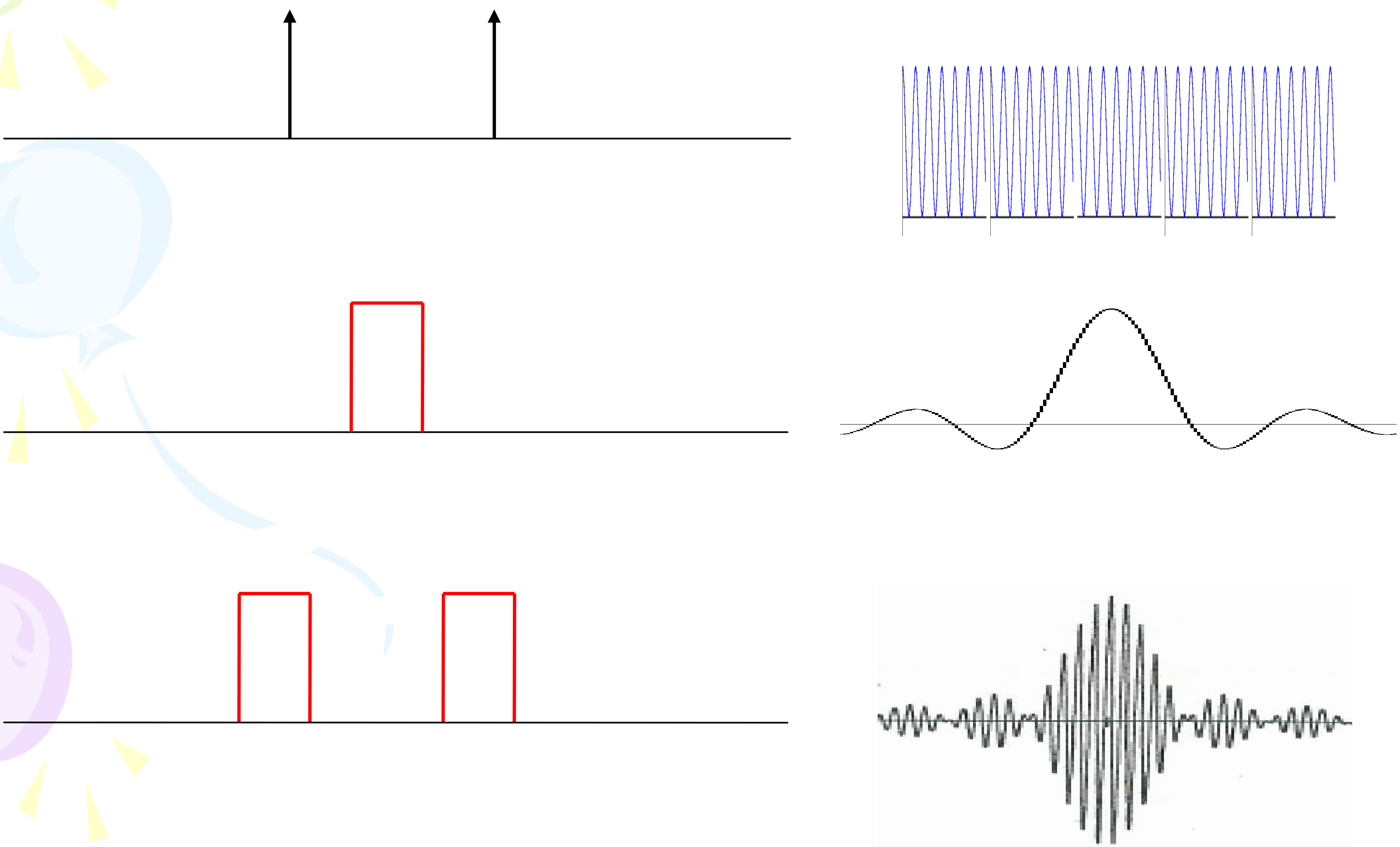
(...so by way of a review, here are voltage & power beams)

- Things to notice:
- Voltage (sinc) is wider than power ($sinc^2$) by 36%
- Zero points are the same
- Power sidelobes are all positive (of course) and lower
- Convolve beam with source

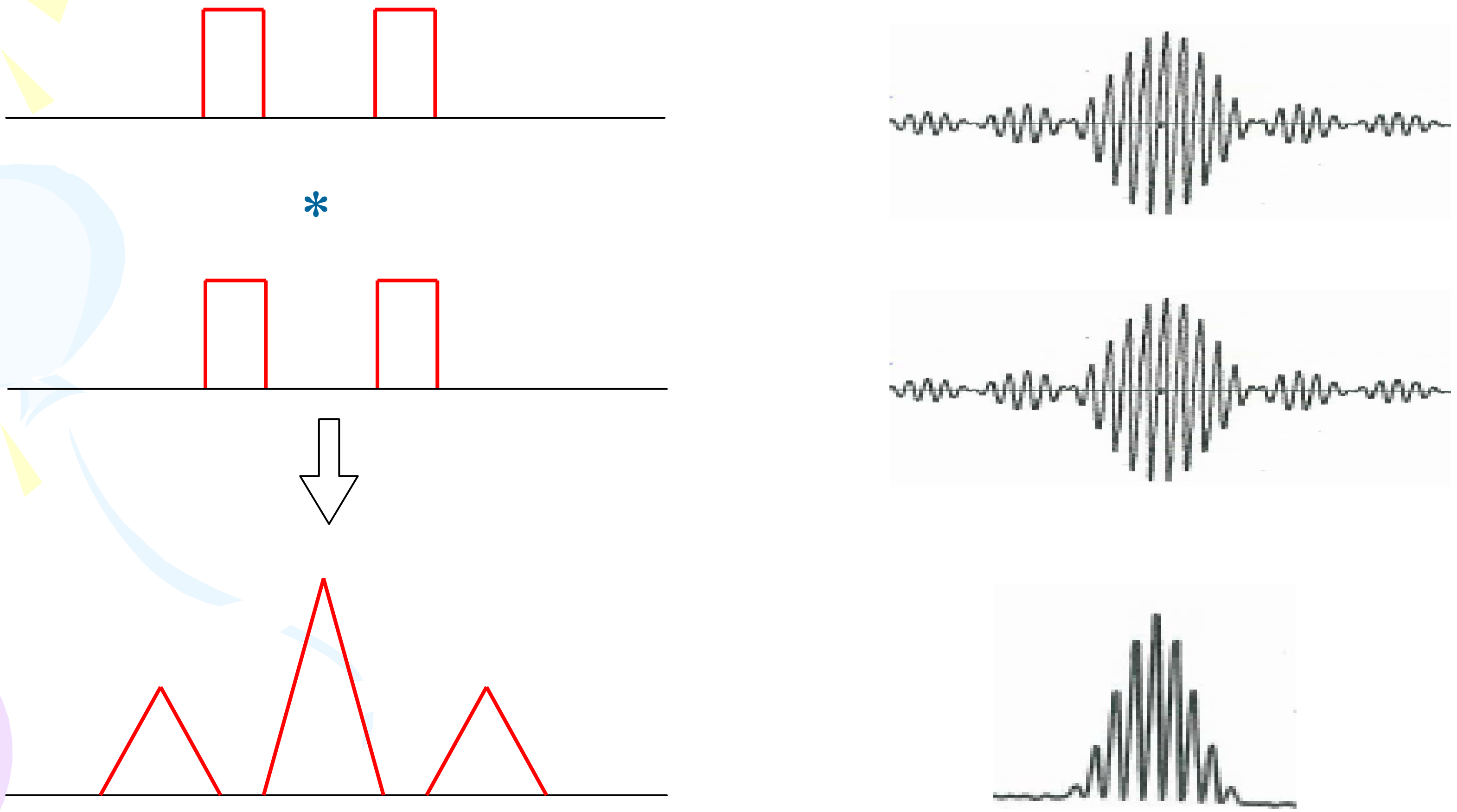




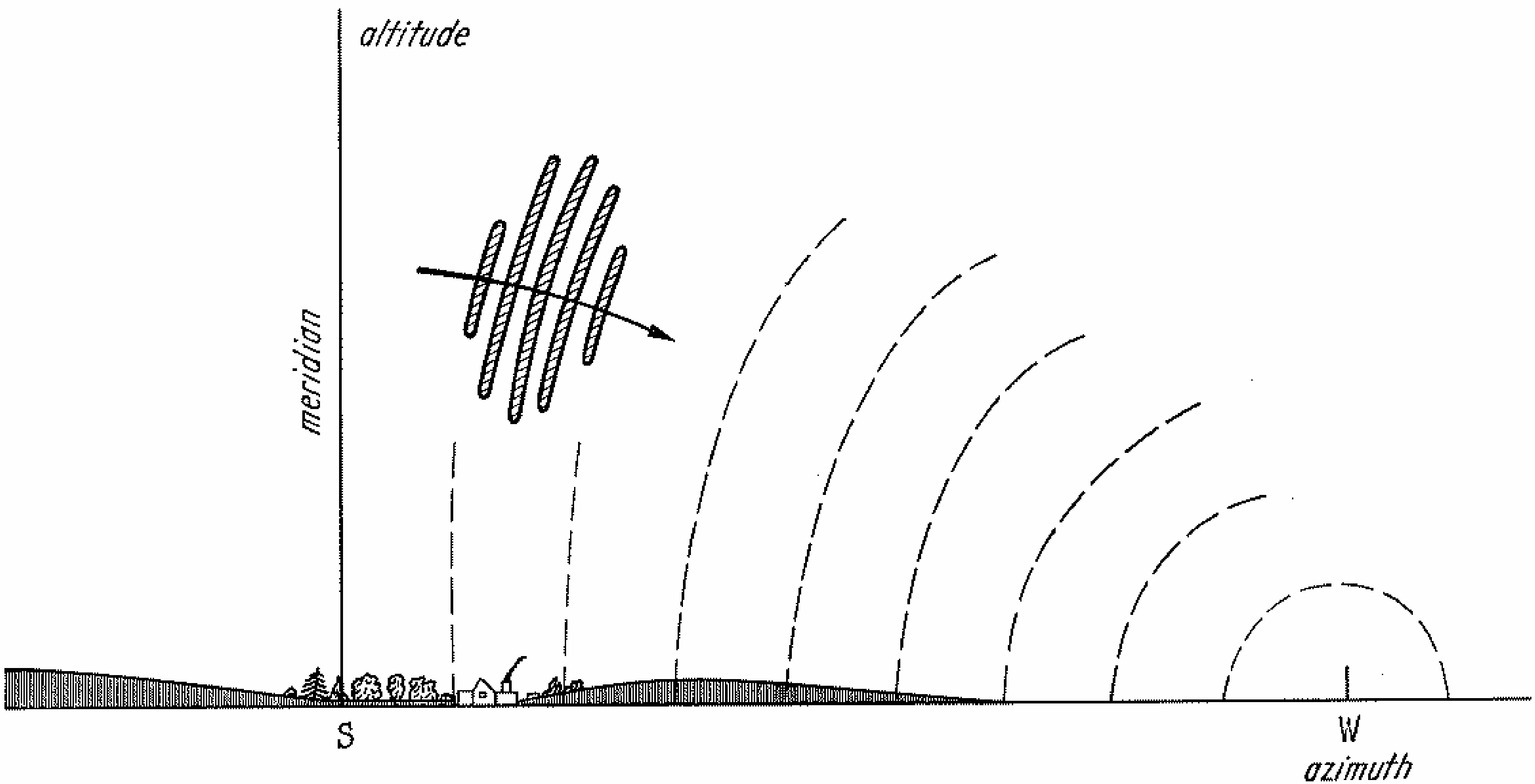
Back to interferometer: we use convolution here, too



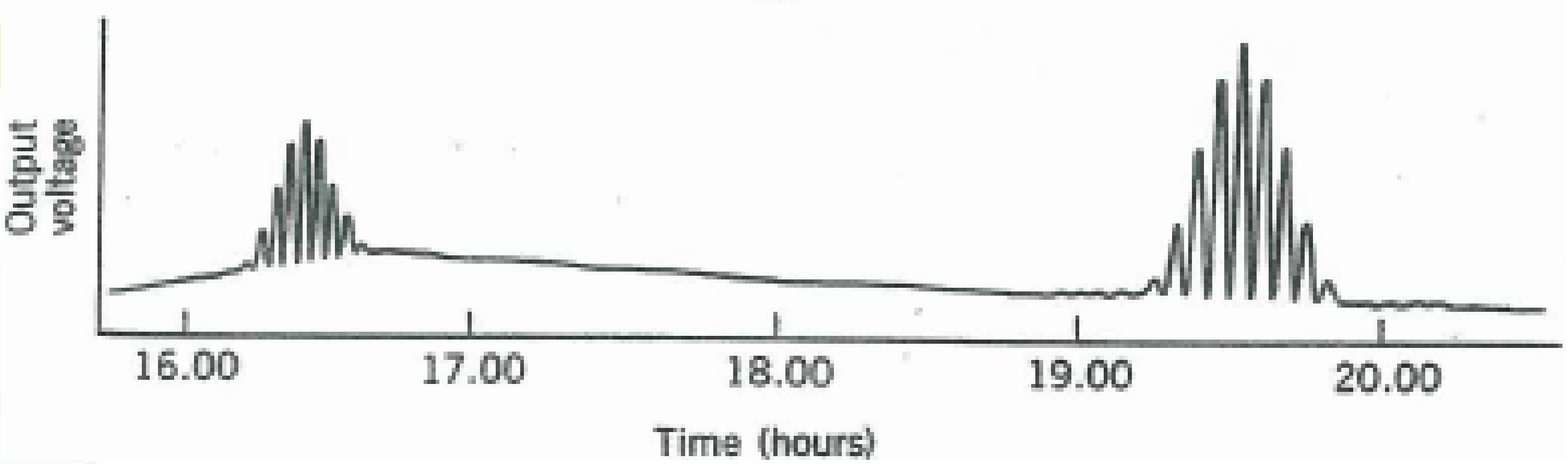
But we still need to derive the power beam pattern



Interferometer sets fringes on sky Baseline determines fringe spacing Element beam picks out region

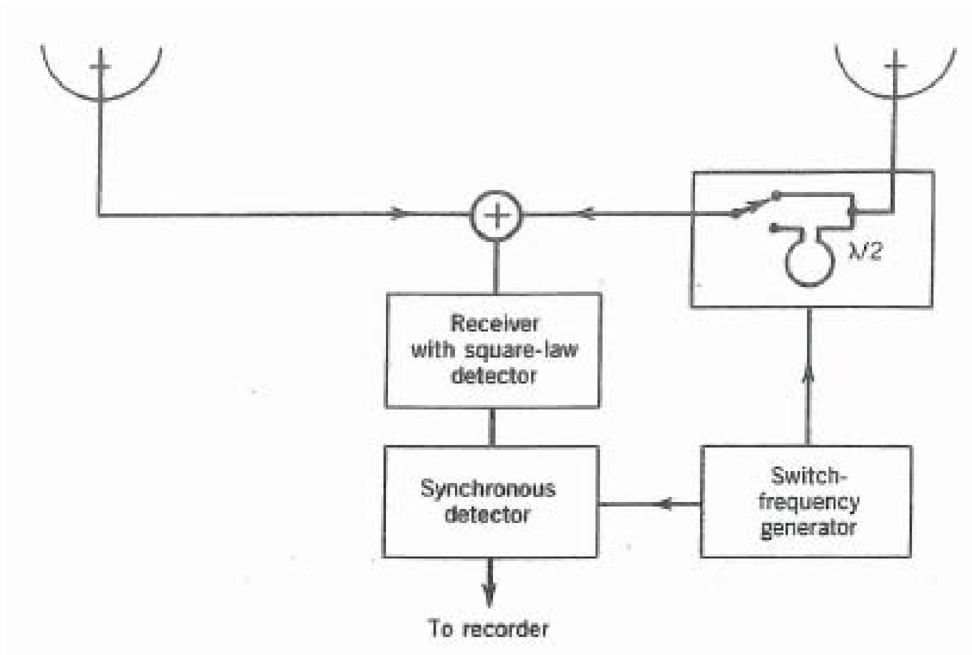


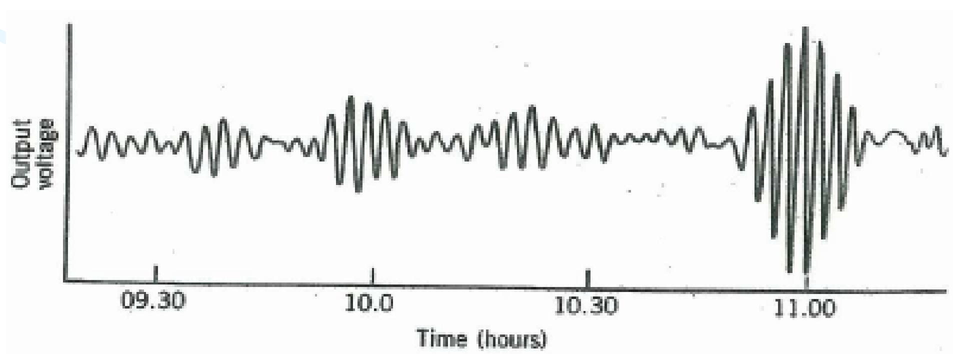
We are now able to explain simple interferometer result



- Observation is convolution of source by beam
- But notice the background level. This is because we have both the interferometer and single dish (central triangle) response
- This background, which may vary, is not always desirable, but it can be eliminated

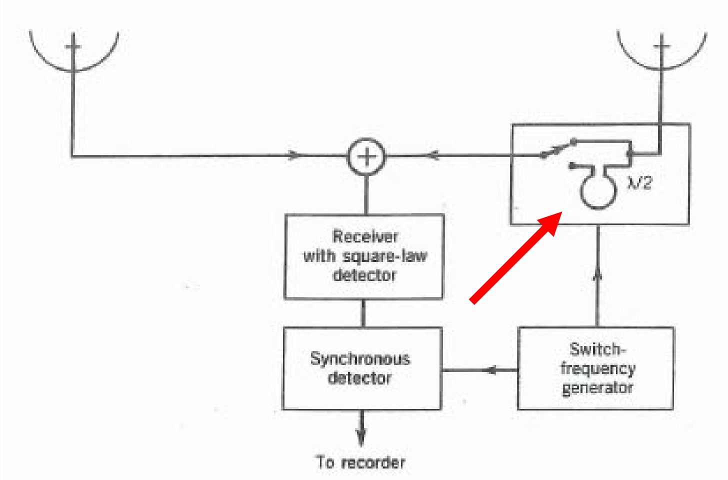
Martin Ryle's interferometer and its response



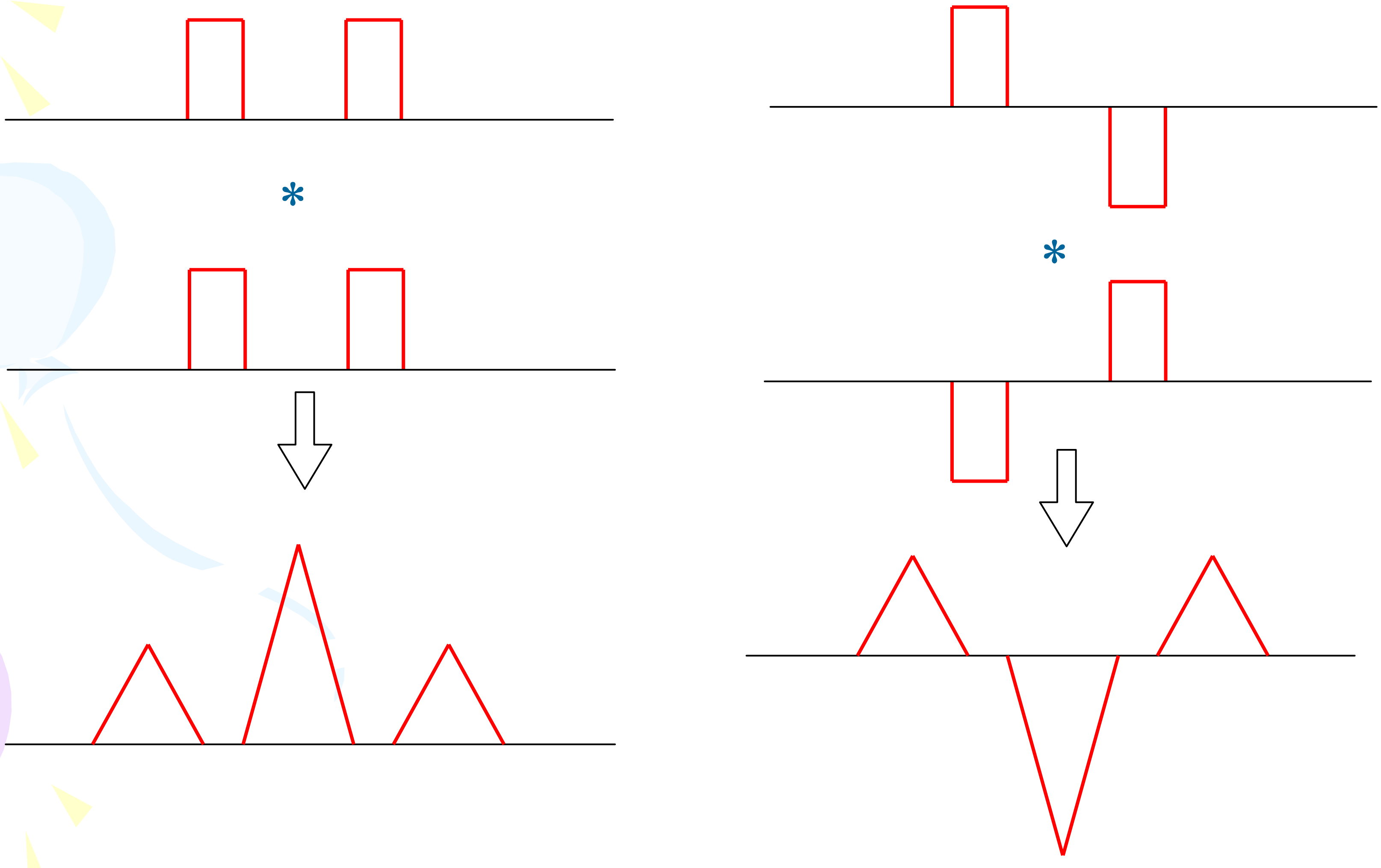


What is effect of introducing 180º (λ/2) phase switch?

- It is introduced into one arm of interferometer
- It has the effect of reversing the signal (multiply: $\times$ -1)
- Remember, to get the instrument's response, we have to convolve the aperture distributions
- Convolution means reversing one function

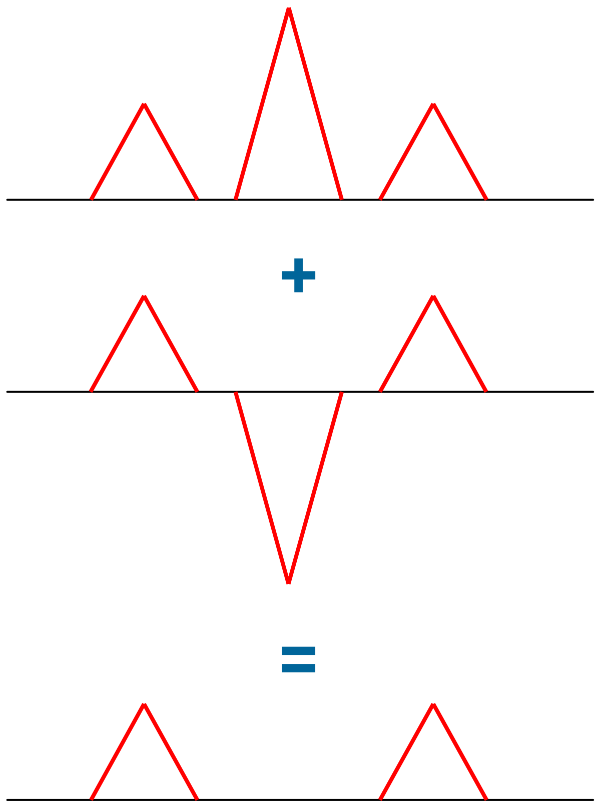


What Ryle did was clever – a bit like Dicke's switch

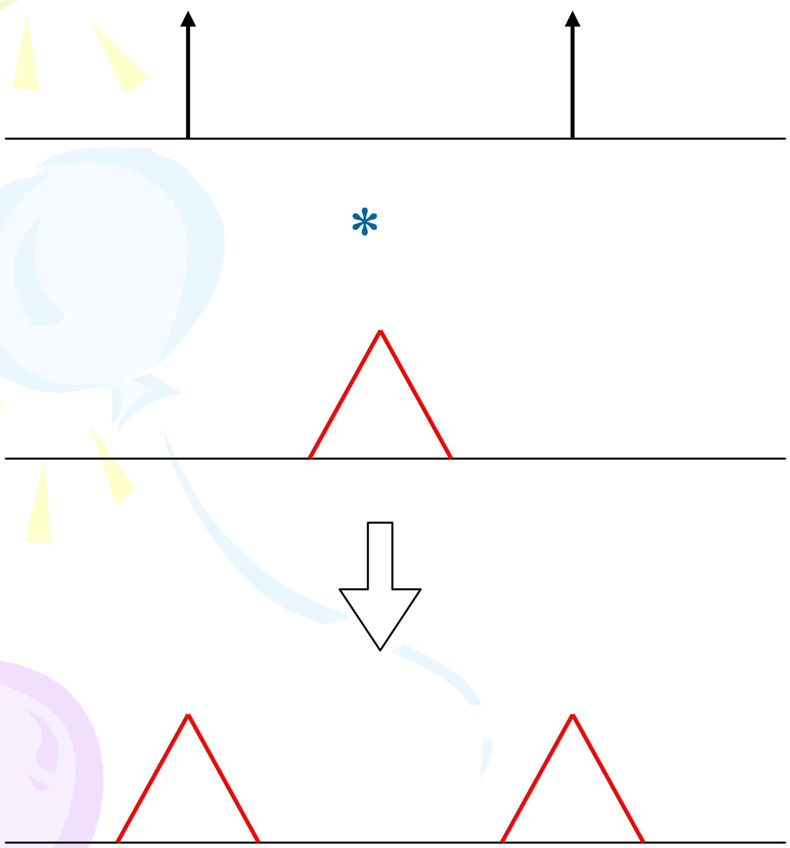


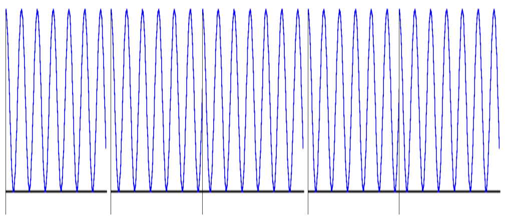
By combining the two, we can eliminate the center triangle

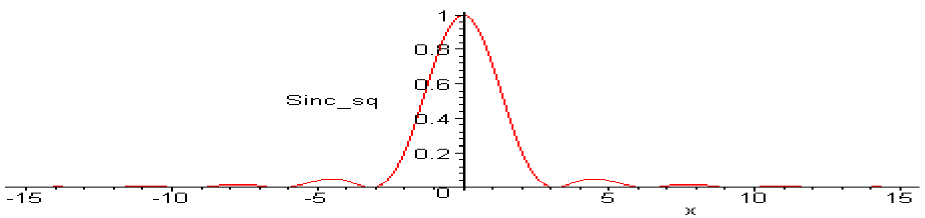
- The center triangle is just the single dish response
- The outer triangles give us the pure interferometer response
- We now need to know its beam

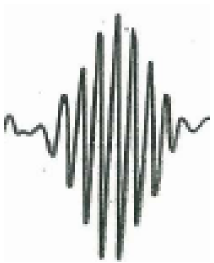


We can determine the beam in the usual way

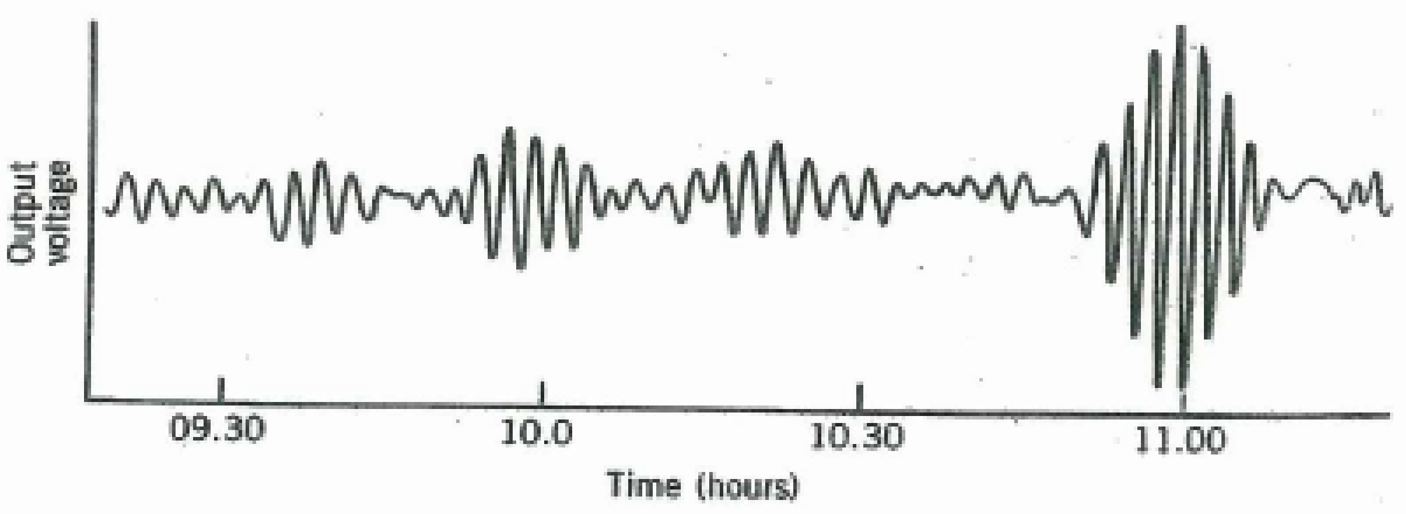








And this accounts for the response in an observation



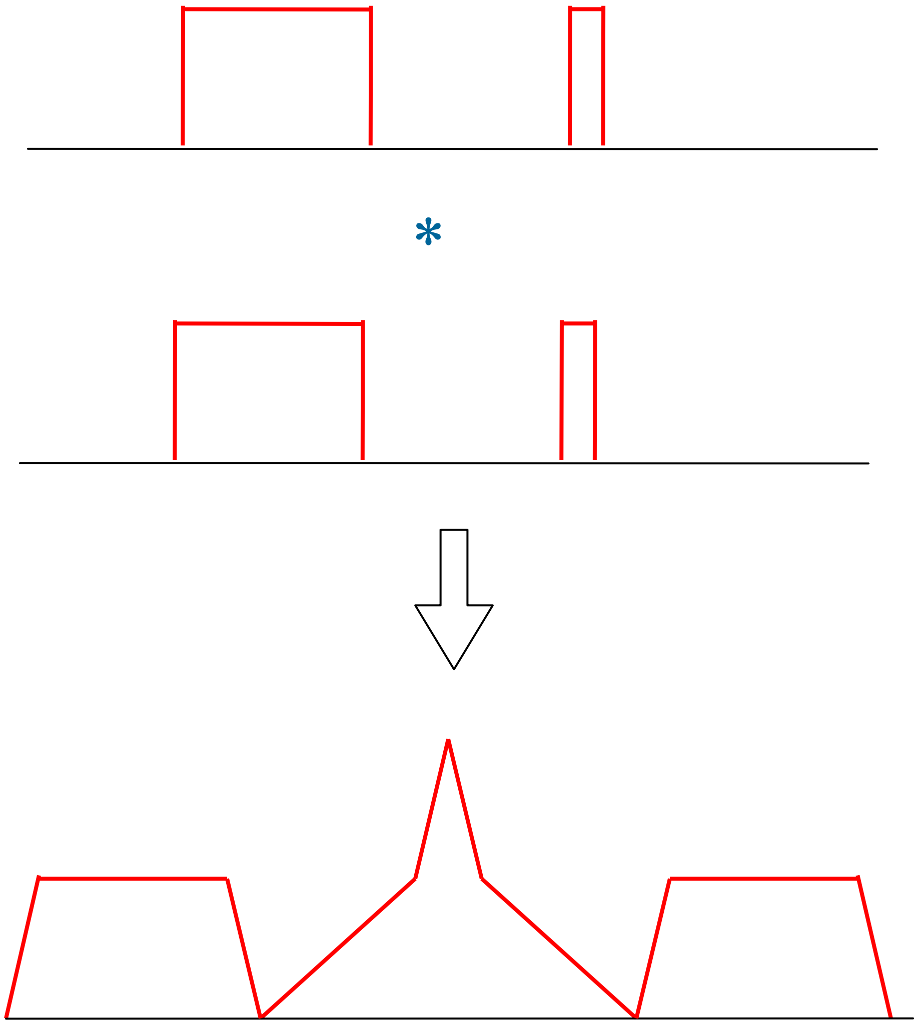
- Result is a flat zero level
- Each squiggly bit is a source convolved by the beam...

...or a sidelobe response (also part of the beam!)

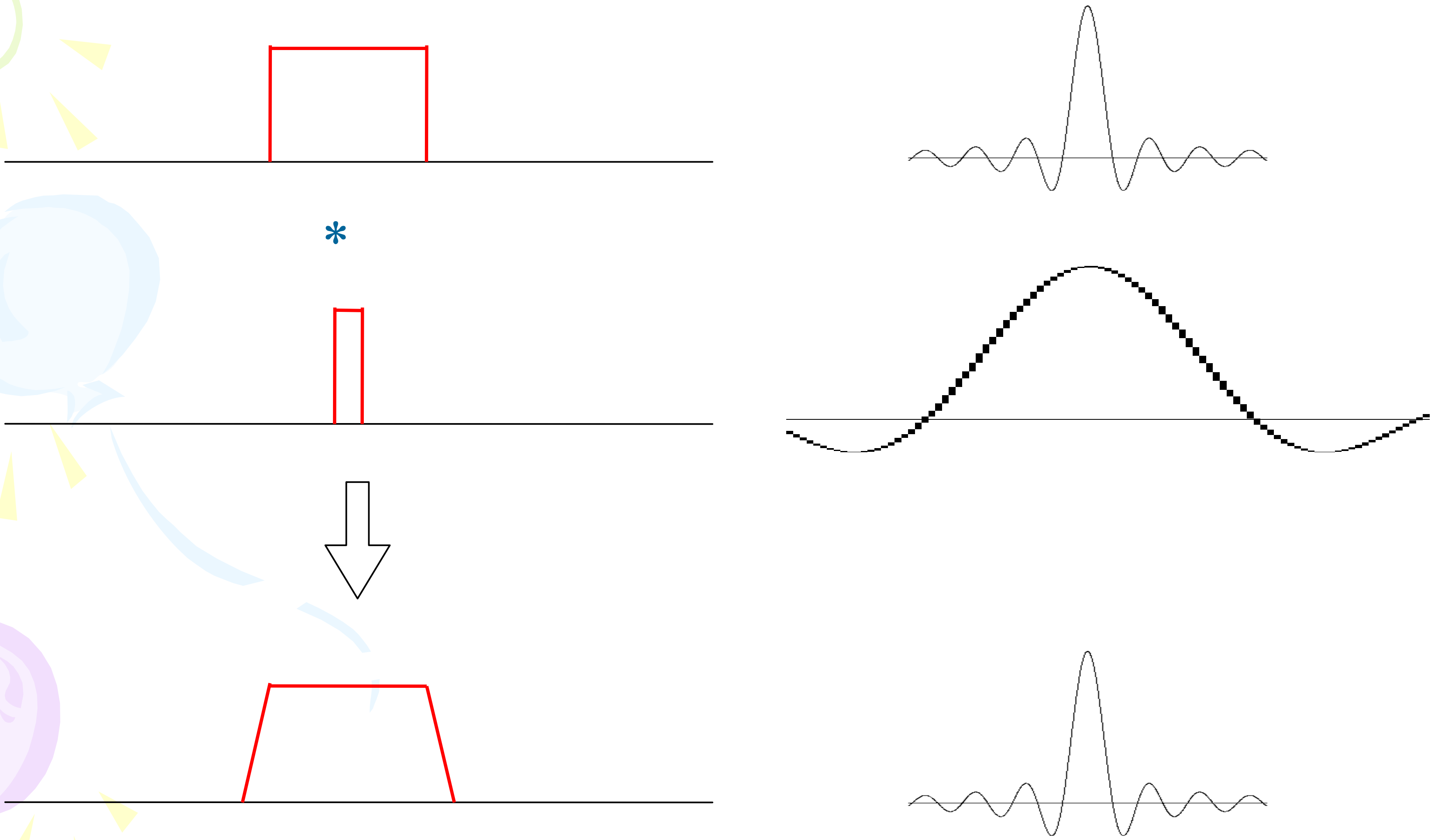
- In fact, this looks near confusion level

What happens if the two elements are not the same?

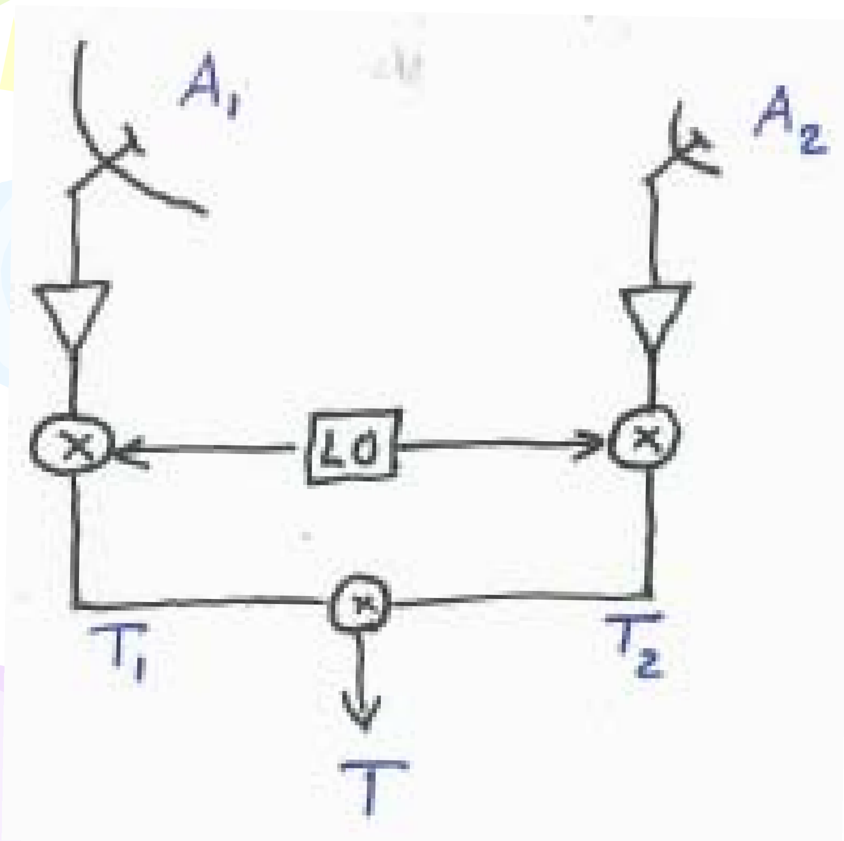
- We can generate, in the usual way, the aperture response
- For the interferometer, the trapezoids, right and left, determine the beam



For very unequal elements, get nearly voltage of large one



Interferometer – unequal elements & sensitivity

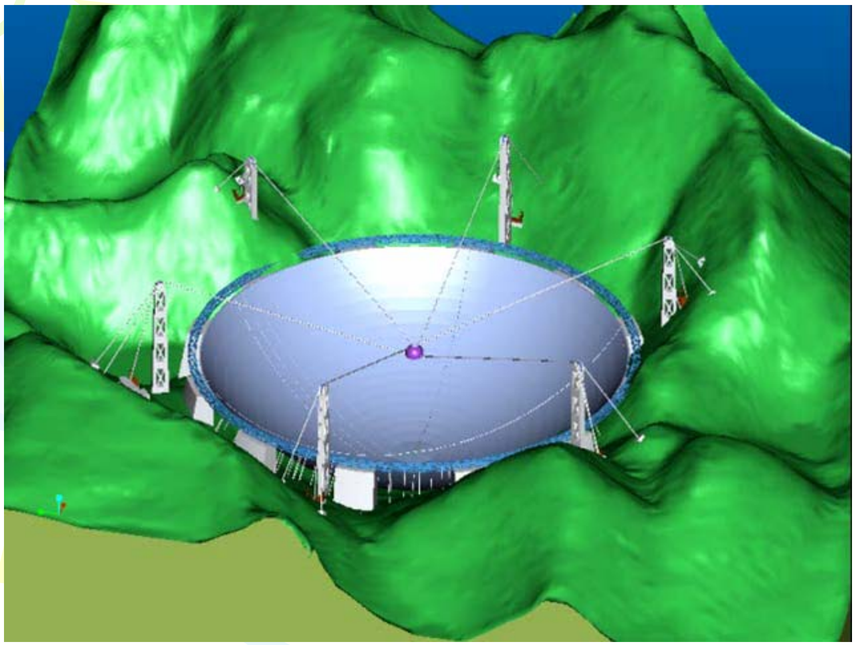


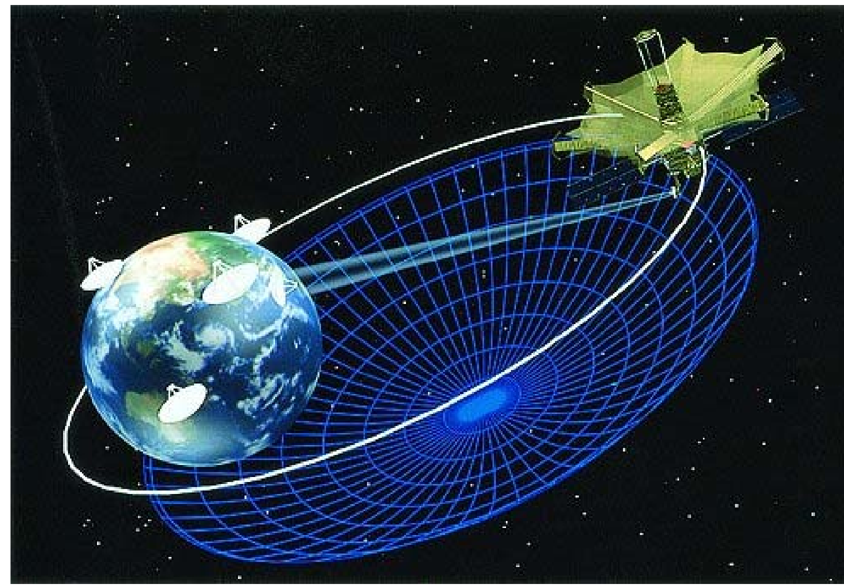
Where A_e and T_s of the elements are unequal, the interferometer values can be simply calculated:

$$A_{int} \approx (A_1 \times A_2)^{1/2}$$

$$T_{int} \approx (T_1 \times T_2)^{1/2}$$

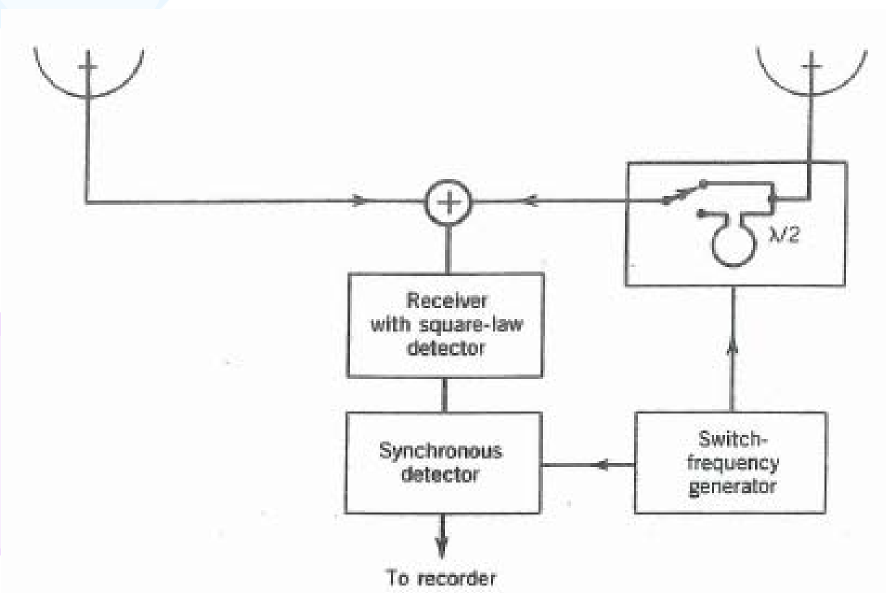
Can have particular advantage in, for example, space VLBI

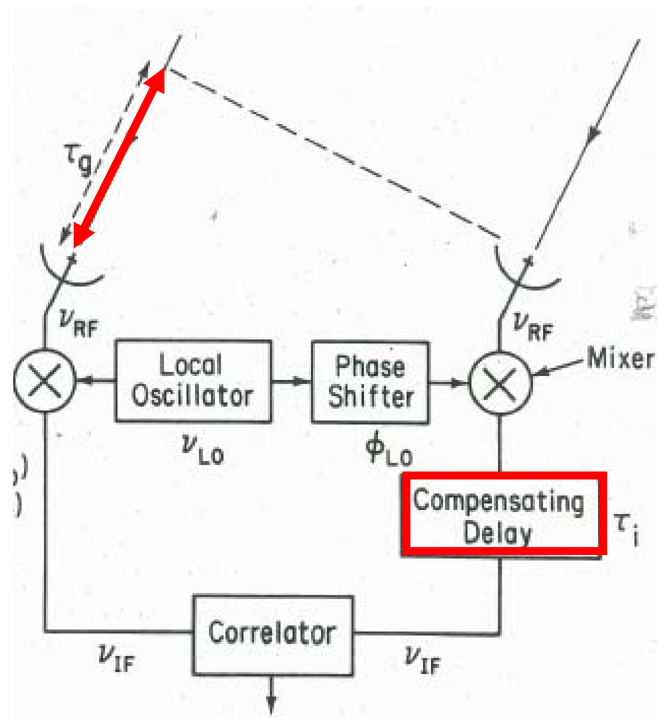




- Combining FAST (300 m) with VSOP (10 m) gives equivalent of: $(300\ \text{m} \times 10\ \text{m})^{1/2}$ = 55 m dish
- Probably cheaper than putting 55 m dish in space

Early interferometers like Ryle's only observed sources at transit. Observing all over the sky requires delay correction to avoid decorrelation.





Effect of delay on interferometer

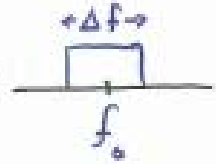
$$S \cdot A^2 \xrightarrow{F.T.} r$$

(S: source; A: antenna; r: response)

Delay (τ): [NB $\tau \sim \frac{1}{f}$]

$$r(\tau) = \iint S(l,f)\, A^2(l,f)\, F(f)\, e^{-2\pi i f\tau}\, dl\, df$$

(F: filter)

$F(f)$: 

$$F = \begin{cases} 1, & f_0 \pm \frac{\Delta f}{2} \\ 0, & f < f_0 - \frac{\Delta f}{2},\ f > f_0 + \frac{\Delta f}{2} \end{cases}$$

$$b(\tau) = \int F e^{-2\pi i f\tau}\, df \; ; \quad F \xrightarrow{F.T.} b$$

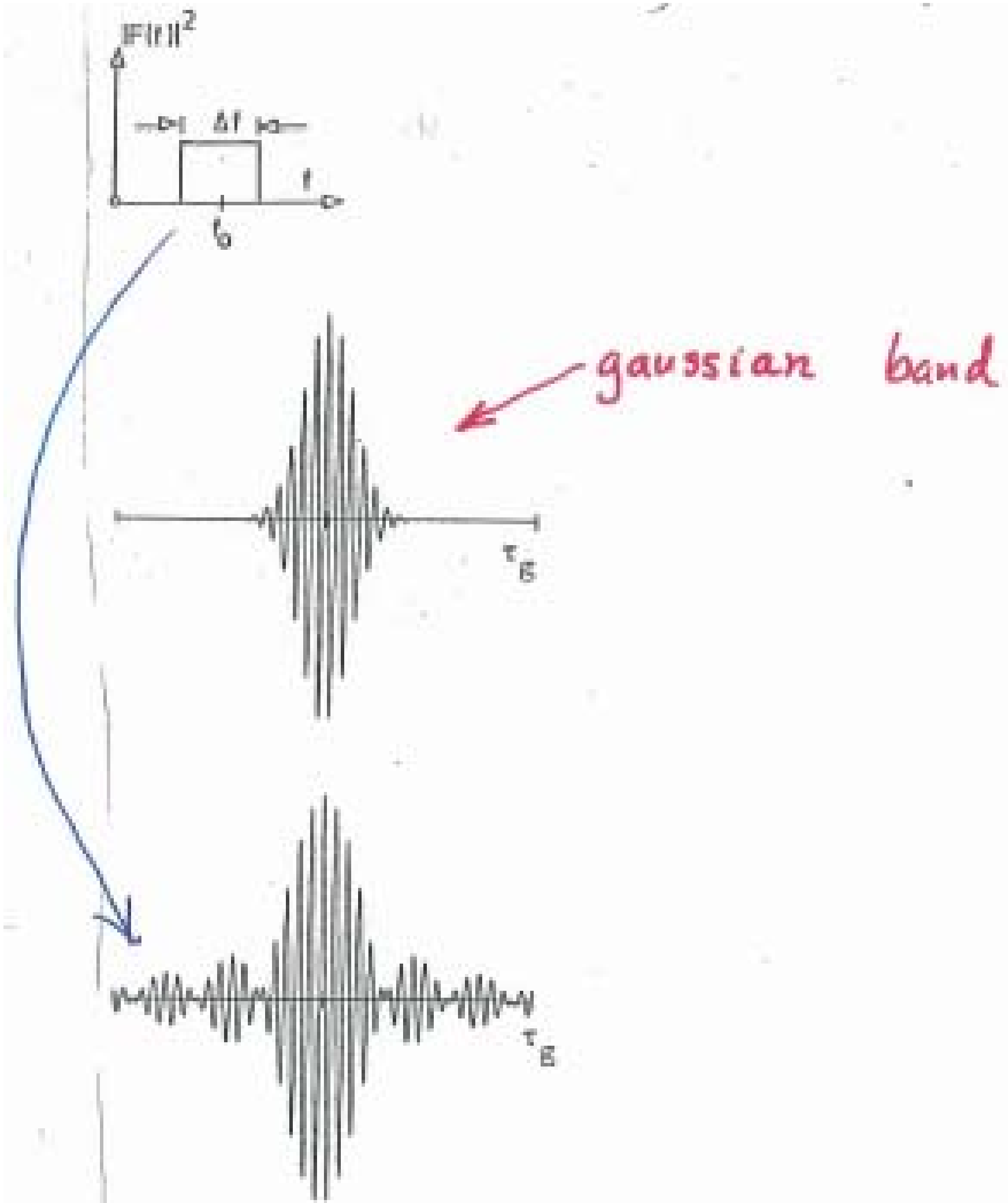
$$b(\Delta f\tau) = \frac{\sin(\pi\Delta f\tau)}{\pi\Delta f\tau} = \mathrm{sinc}(\pi\Delta f\tau)$$

NB: $\pi\Delta f\tau \ll 1$ (if not, decorrelation)

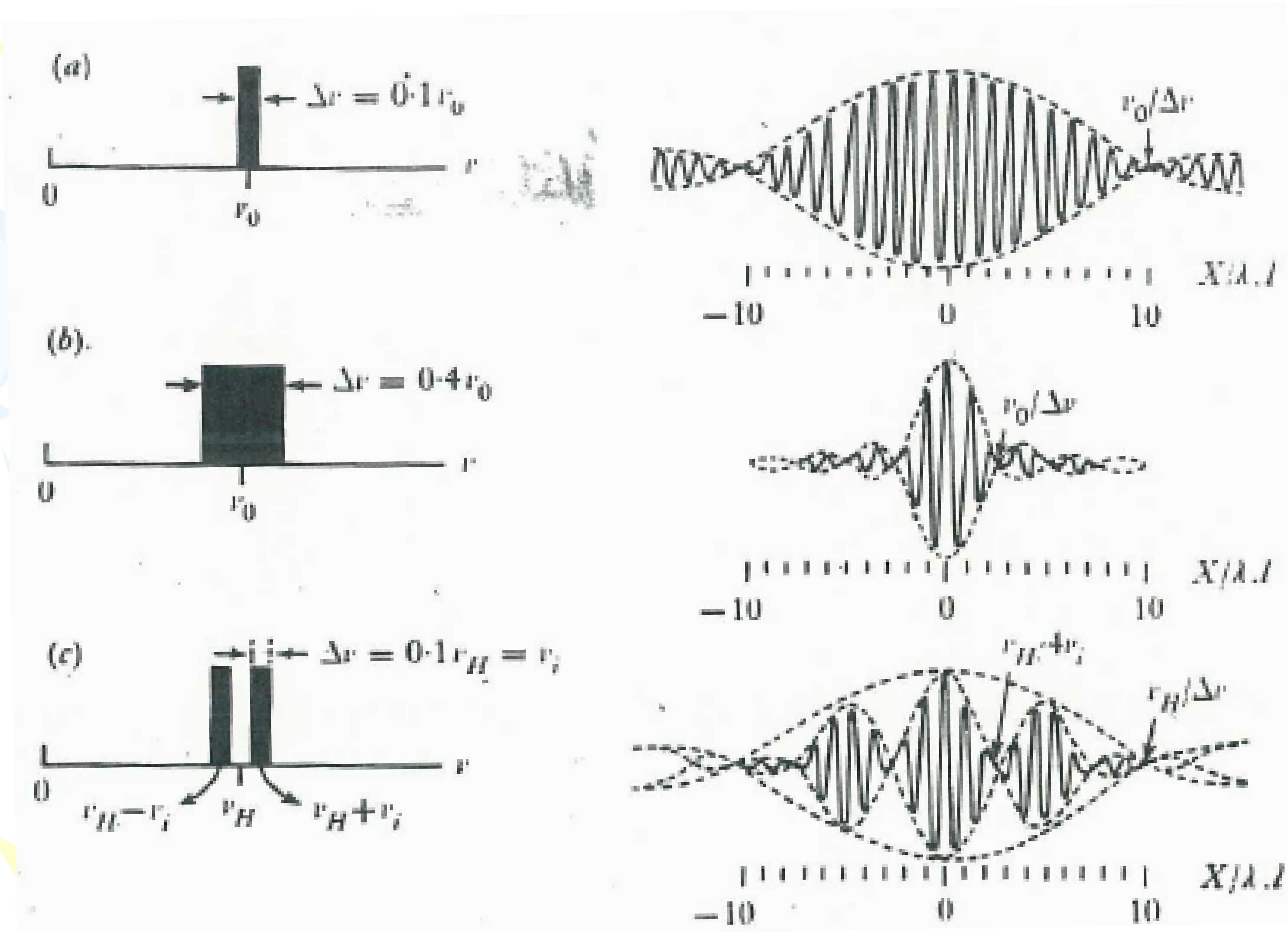
e.g. $\Delta f = 1$ MHz, $\tau \ll 0.3\,\mu$sec

($c\,0.3\mu s \sim 90$ m)

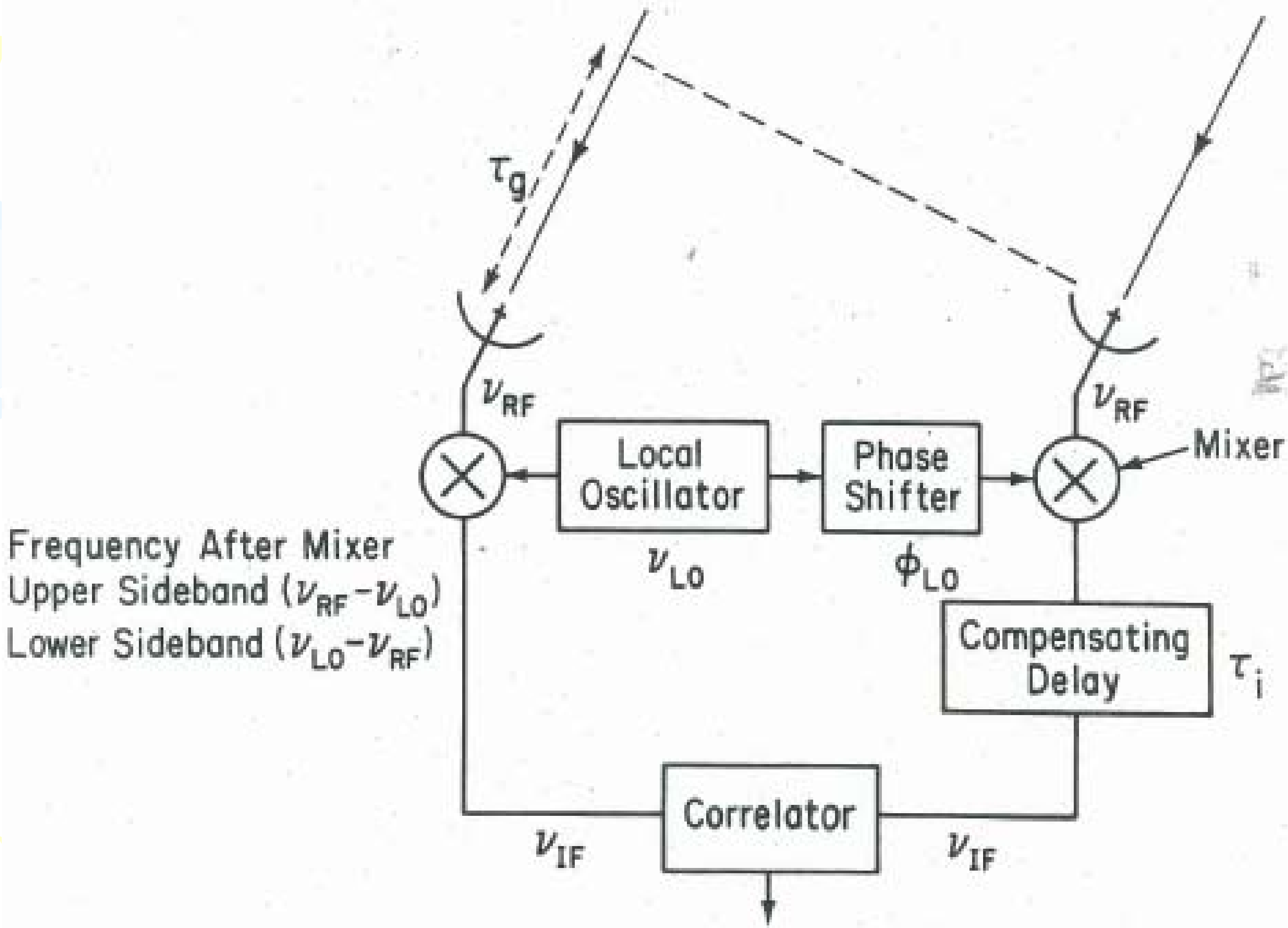
Delay beam response



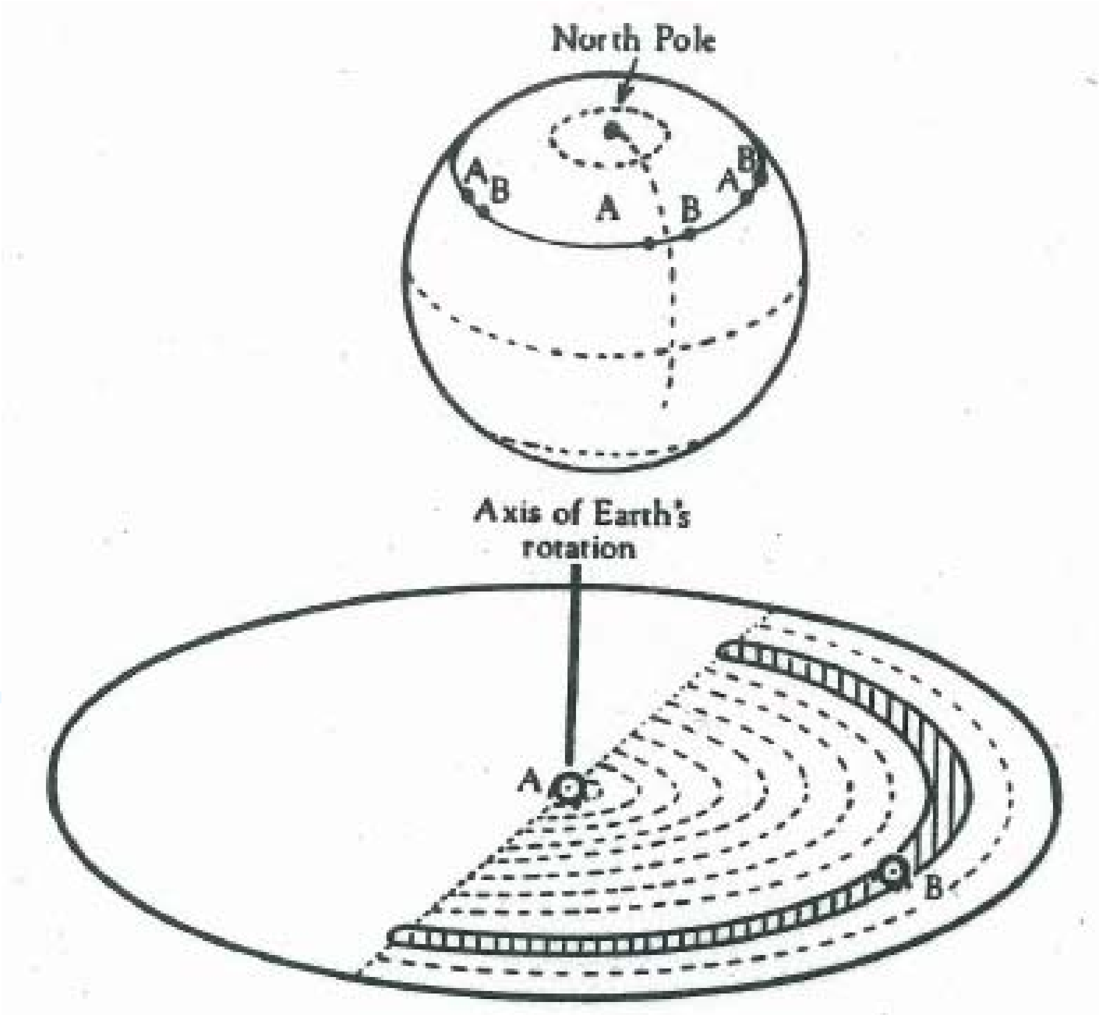
More delay beam responses



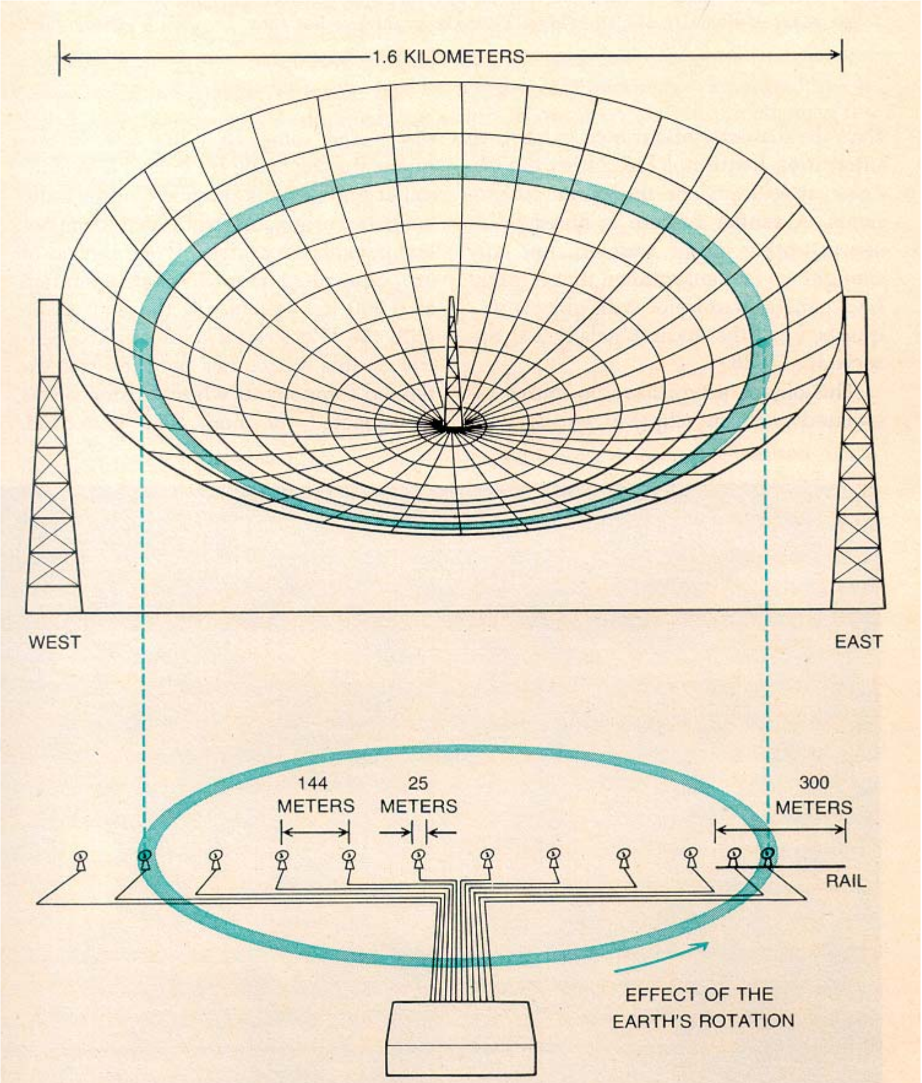
A modern interferometer with delay compensation



Ryle also introduced the idea of earth rotation synthesis



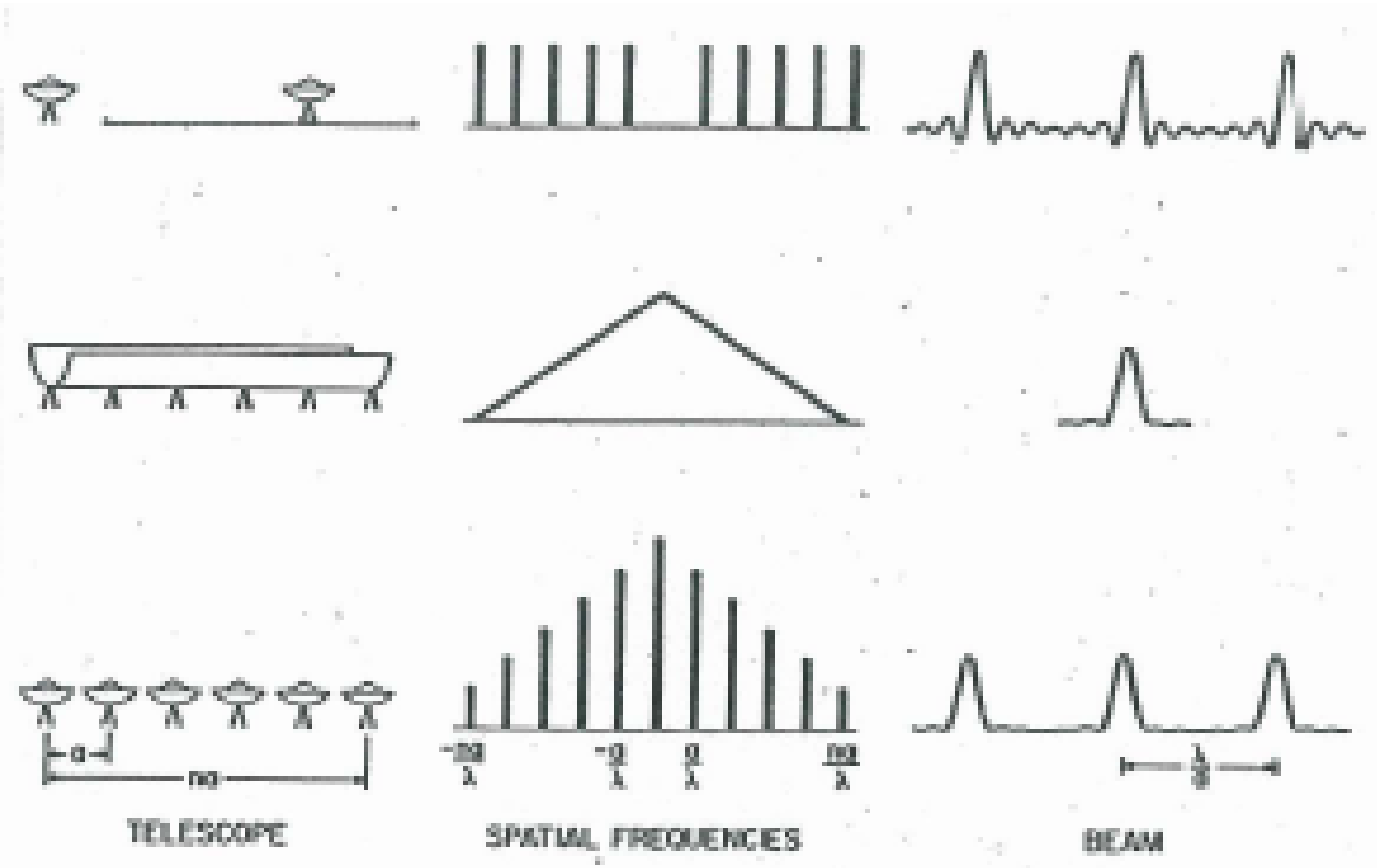
Also called super synthesis: synthesize large antenna



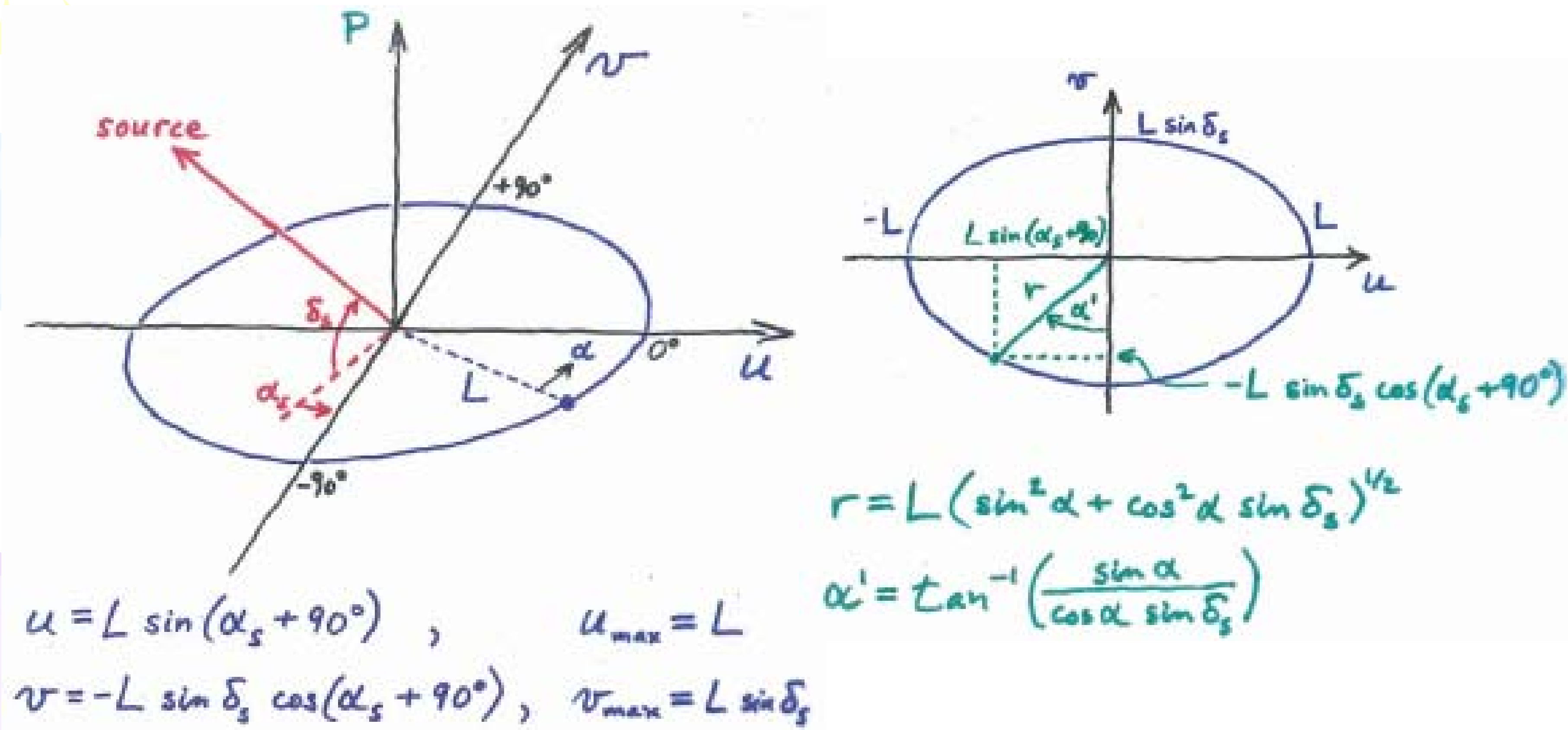
Use several small dishes. Move dishes and re-observe.

WSRT: more dishes, faster. Principle: sources not vary.

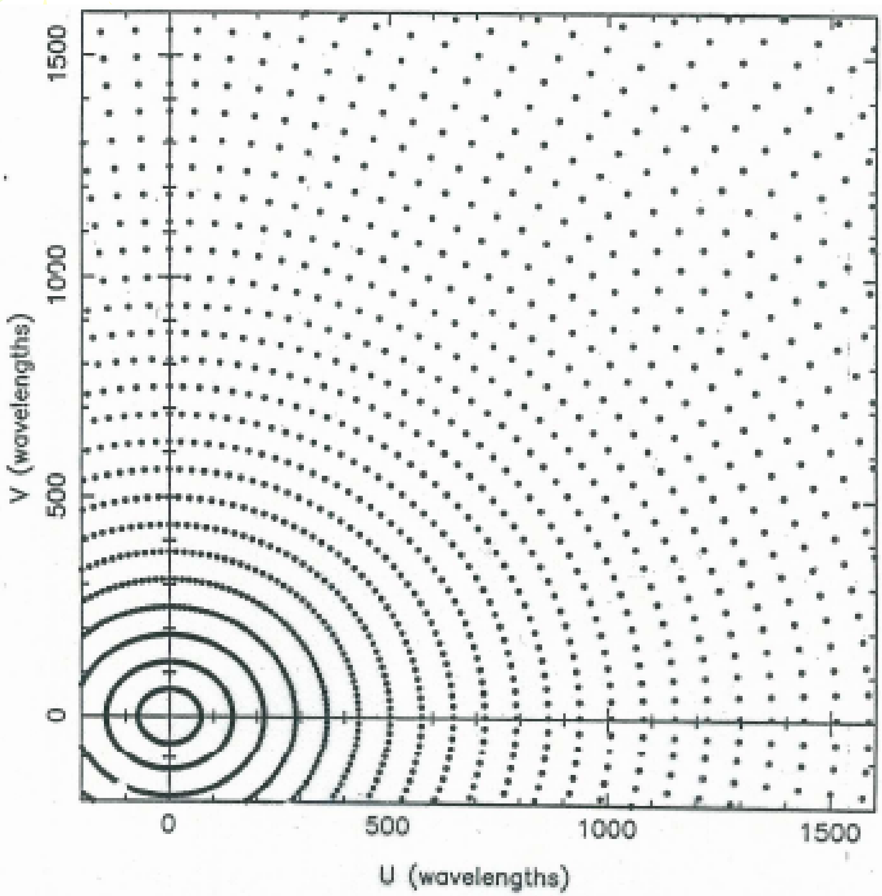
Beam of several regular arrays in one dimension

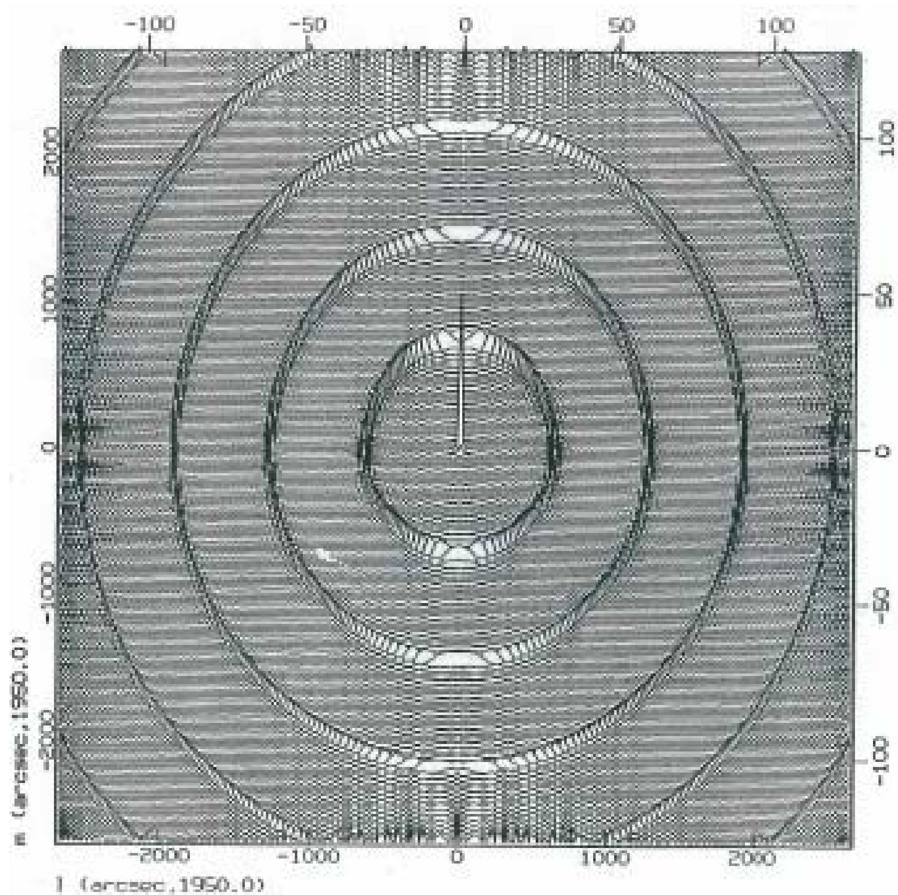


Geometry of east-west array and the u,v-plane

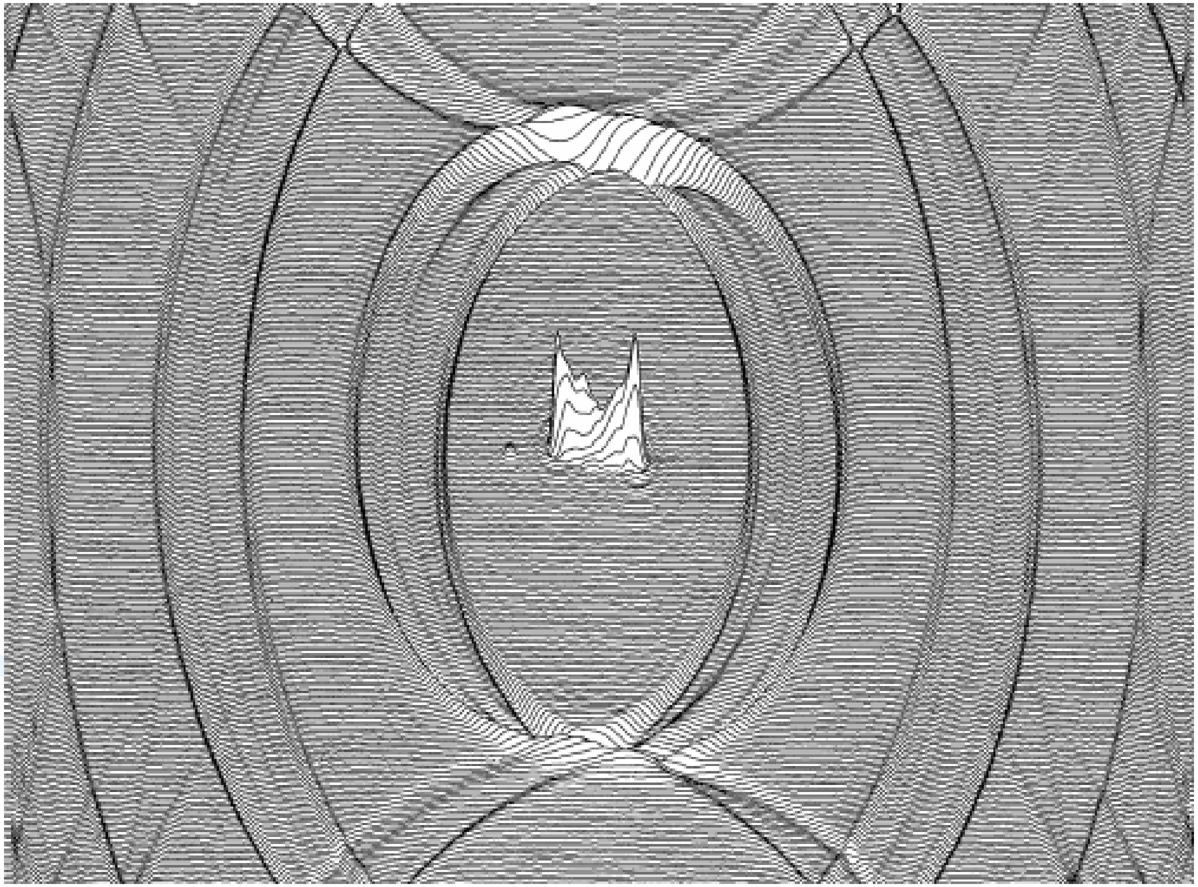


Example of WSRT u,v-ellipses, and the antenna pattern

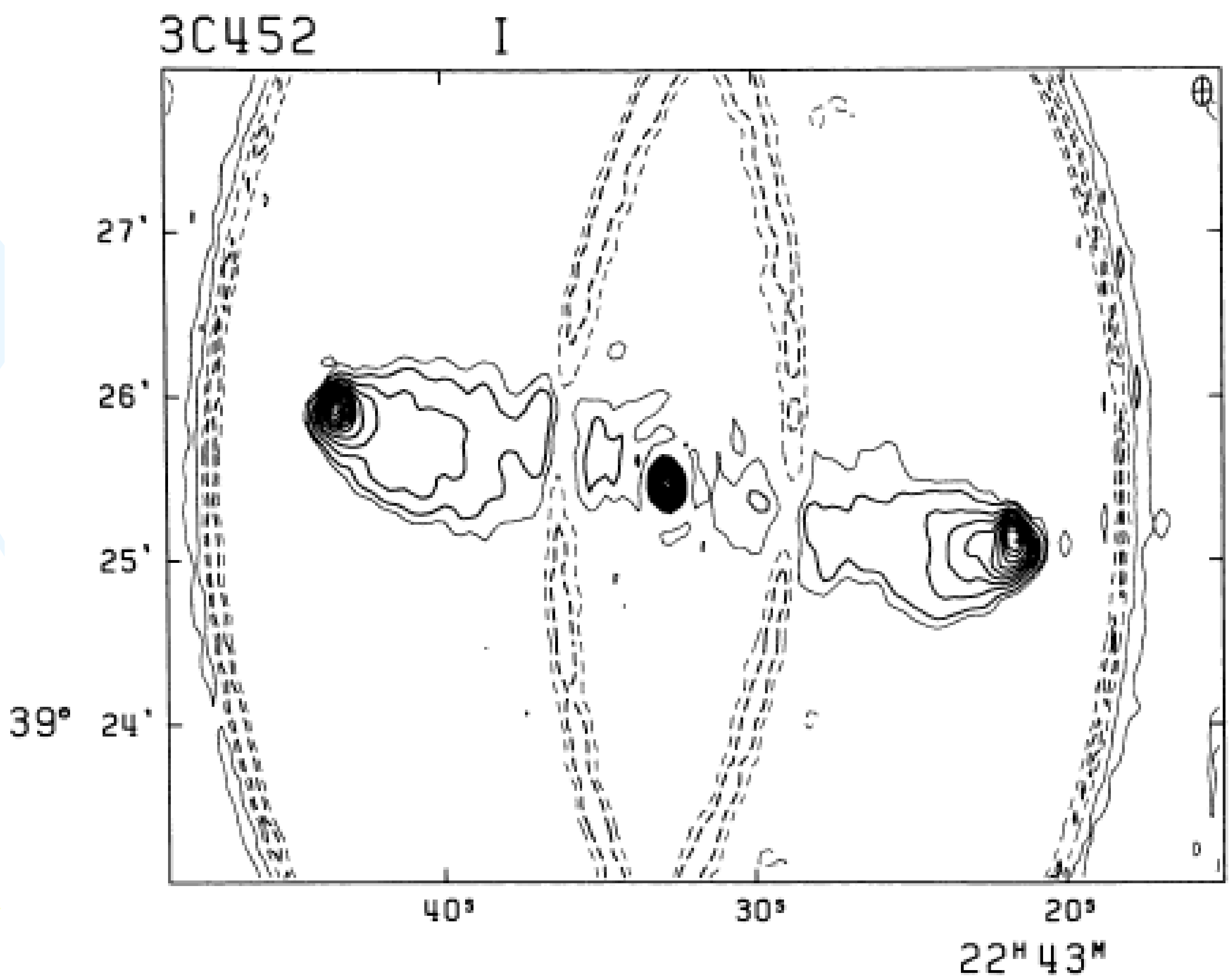




The source is convolved with the whole beam



Source size < grating lobe size - an example of self confusion



An important property: Hermitian symmetry

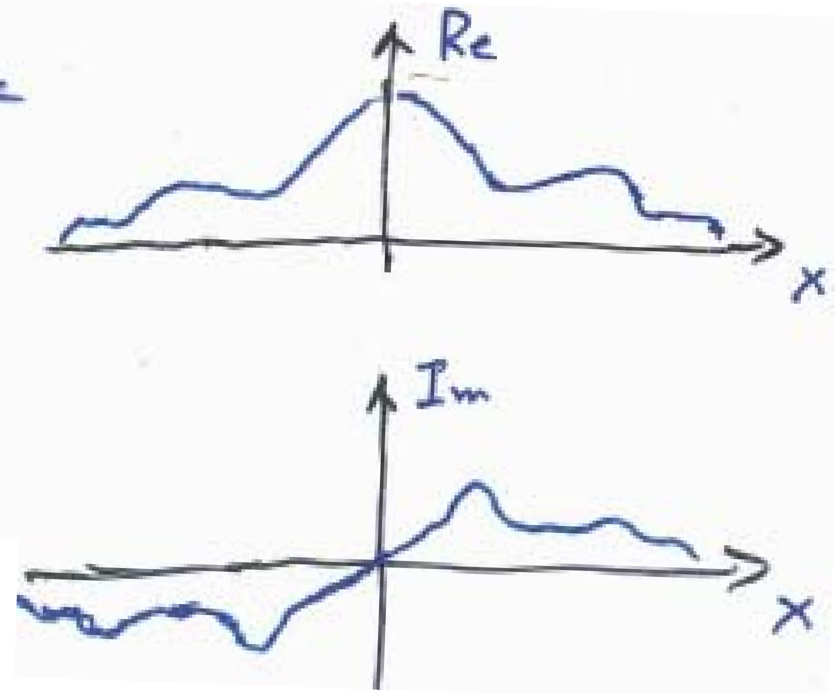
The sky brightness is real

Its Fourier Transform is ∴ Hermetian symmetric

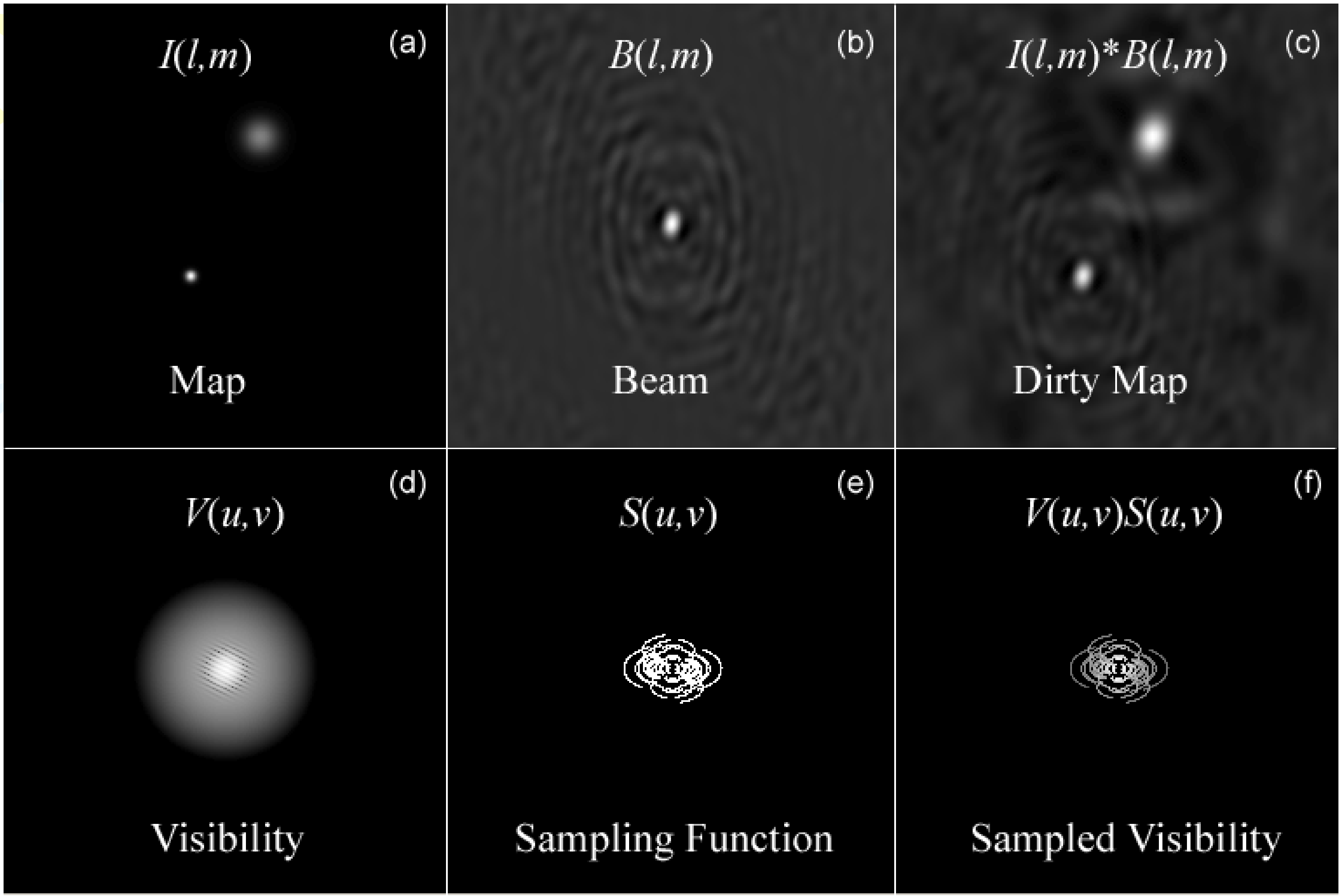
$I(\theta) \xrightarrow{FT} i(x)$; $I(\theta)$ ~ real

$i(x) = i_R(x) + i_I(x)$

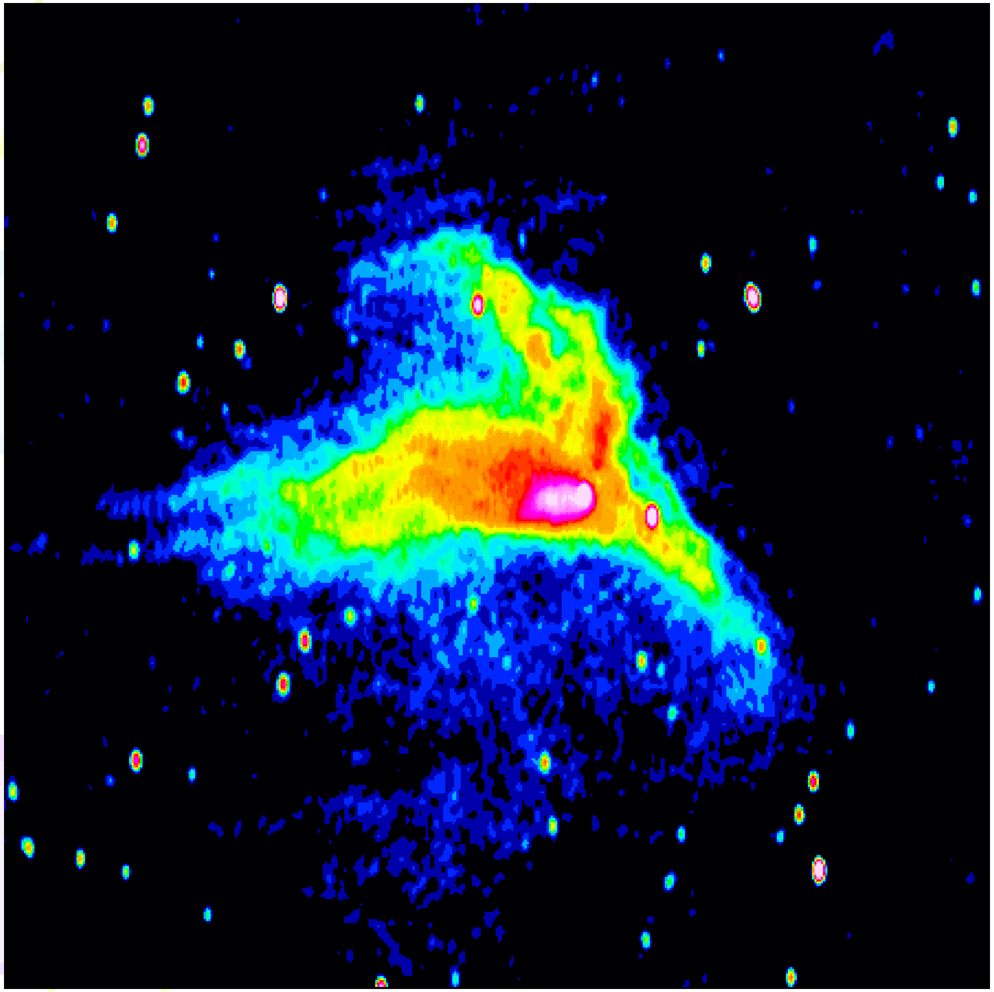
$i(-x) = i_R(x) - i_I(x)$

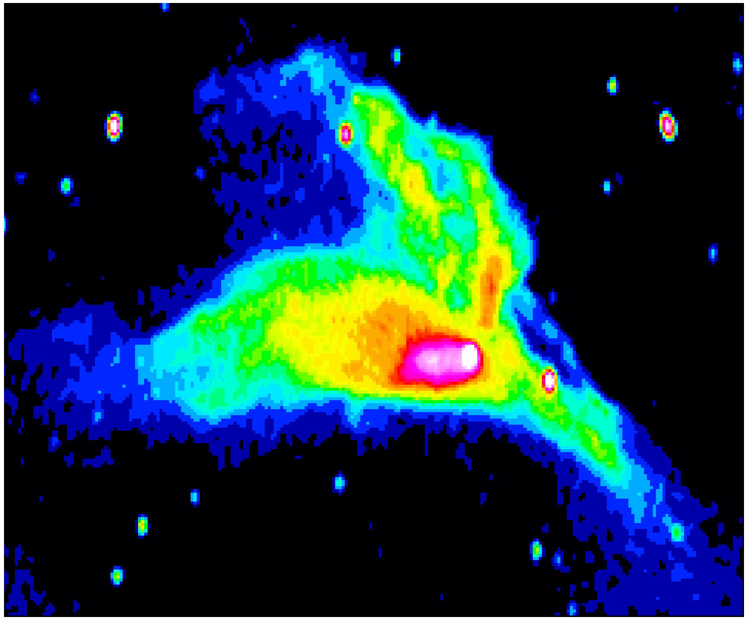


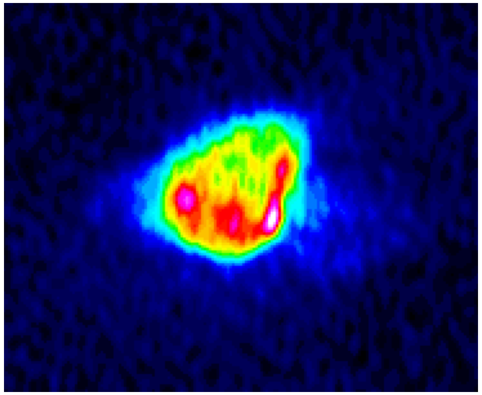
Observation of two sources with interferometer array



WSRT observation of CTB80 at 92 cm, 49 cm & 3.6 cm







WSRT and VLBI observation of giant radio galaxy 3C236